













Making Sense of Dependence: Efficient Black-box Explanations Using Dependence Measure

PAUL NOVELLO, THOMAS FEL, DAVID VIGOUROUX, NEURIPS 2022









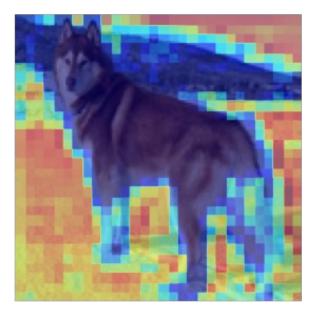


Introduction – why explainability in Deep Learning?



- o Build trust in the model prediction
- O Make sure the model makes a prediction for a good reason
 - Identify bias or spurious effects learned by a model





Introduction – why explainability in Deep Learning?



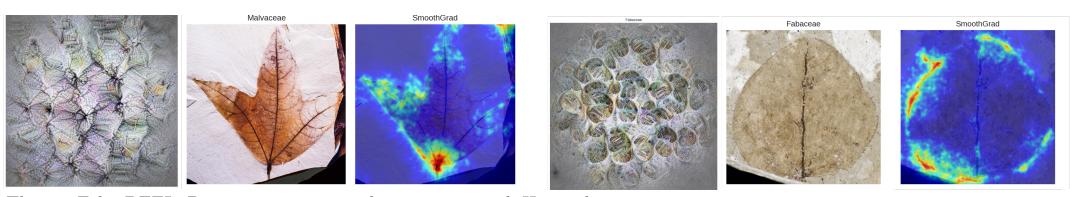
- o Build trust in the model prediction
- O Make sure the model makes a prediction for a good reason
 - Identify bias or spurious effects learned by a model
 - Understand failure cases



Introduction – why explainability in Deep Learning?



- o Build trust in the model prediction
- O Make sure the model makes a prediction for a good reason
 - Identifiy bias or spurious effects learned by a model
 - Understand failure cases
- o Pattern mining: identify patterns in data



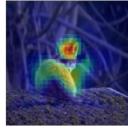
Thomas Fel – DEEL, Brown university, work in progress with Harvard university

A zoology of attribution methods

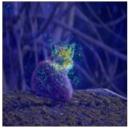


Vulpes vulpe

Occlusion sensitivity



Saliency



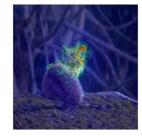
Integrated Gradients



DeconvNet



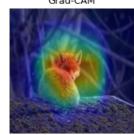
SmoothGrad



Guided Backpropagation



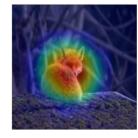
Grad-CAM



Gradient x Input



Grad-CAM++



A zoology of attribution methods



Saliency Maps Symonyan & al (2013)[1]

$$\Phi = \nabla f(x) \implies \phi_i = \frac{\partial f(x)}{\partial x_i}$$

In an infinitesimal neighborhood (often not feasible), what are my features that most impact the output score?

SmoothGrad Smilkov & al (2017)[2]

$$\Phi = \mathbb{E}_{\epsilon \sim \mathcal{N}(0, I\sigma)} [\nabla f(x + \epsilon)]$$

$$\Phi = \frac{1}{N} \sum_{i=0}^{N} \nabla f(x + \epsilon)$$

$$\Phi = \frac{1}{N} \sum_{i=0}^{N} \nabla f(x + \epsilon)$$

As the name suggests, averages the gradient at several points corresponding to small perturbations around the point of interest.

^[1] Deep Inside Convolutional Networks: Visualising Image Classification Models and Saliency Maps

^[2] SmoothGrad: removing noise by adding noise

A zoology of attribution methods



Integrated Gradients Sundarajan & al (2017)[1]

$$\Phi = (x - x_0) \int_0^1 \frac{\partial f(x_0 + \alpha(x - x_0))}{\partial x} d\alpha$$

$$\Phi = (x - x_0) \frac{1}{N} \sum_{i=0}^N \frac{\partial f(x_0 + \frac{i}{N}(x - x_0))}{\partial x}$$

Averaging the gradient values along the path from a baseline state to the current value. The baseline state is often set to zero.





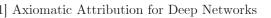




Occlusion Ancona & al (2017)[2]

$$\phi_i = f(x) - f(x_{[x_i = x_0]})$$

Sweep a patch that occludes pixels over the images, and use the variations of the model prediction to deduce critical areas.



^[2] Towards better understanding of gradient-based attribution methods for Deep Neural Networks







A zoology of attribution methods



RISE Petsiuk & al (2018)[1]

$$\phi_i = \mathbb{E}[f(x \odot m)|m = 1]$$

$$\phi_i = \frac{1}{\mathbb{E}[m]N} \sum_{i=0}^{N} f(x \odot m_i) \odot m_i$$

Probing the model with randomly masked versions of the input image and obtaining the corresponding outputs to deduce critical areas.









[1] RISE: Randomized Input Sampling for Explanation of Black-box Models

black-box state of the art

A zoology of attribution methods



And many more ...

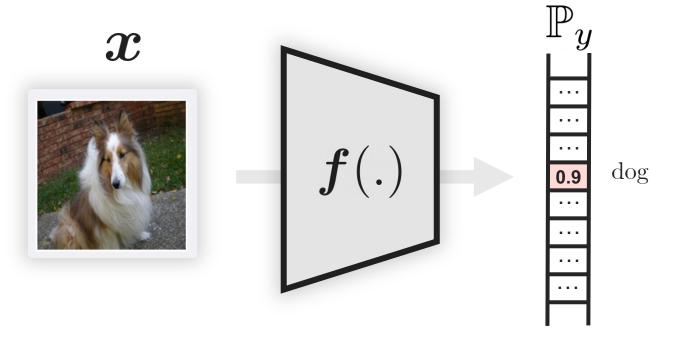
White-box

- Needs access to internal representations
- o Needs a backward pass
- o relatively fast

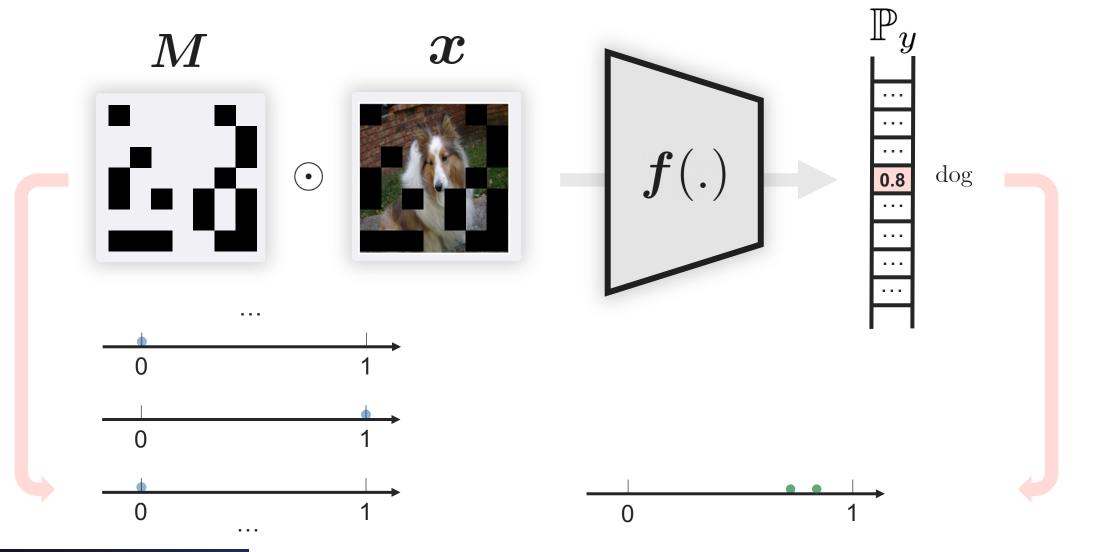
Black-box

- Only needs perturbations on the input space
- o Expensive: many forward passes are required

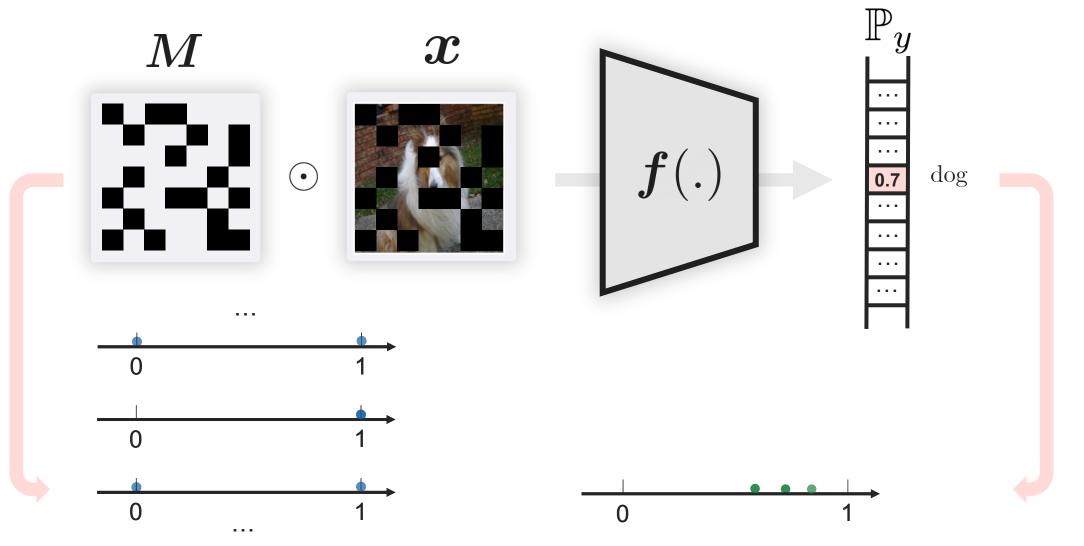




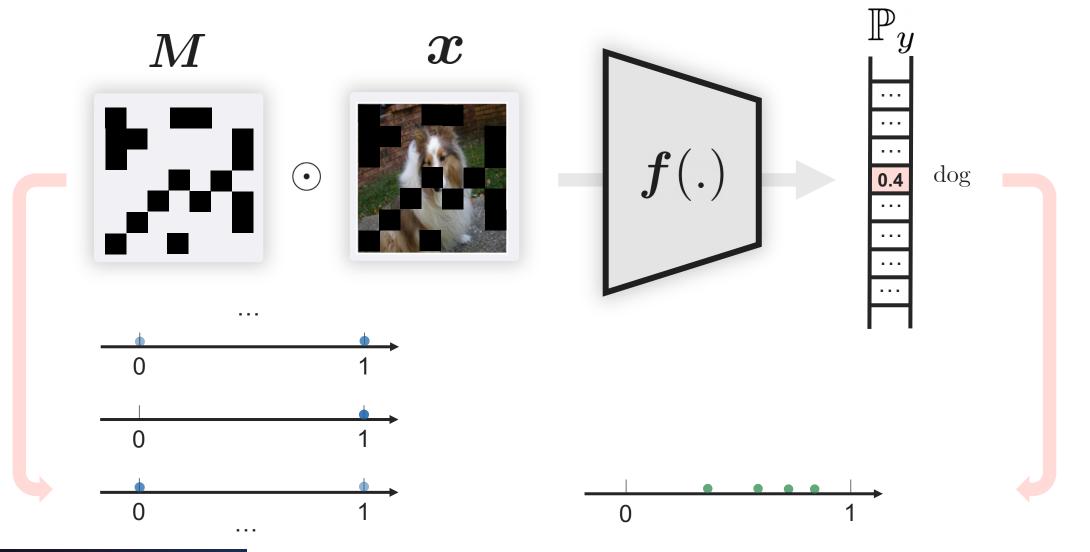




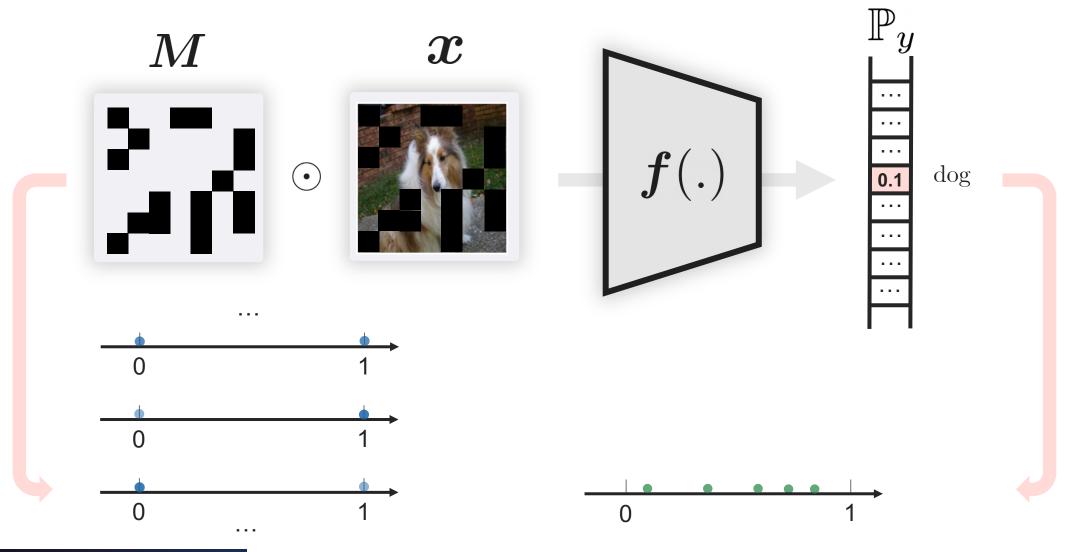




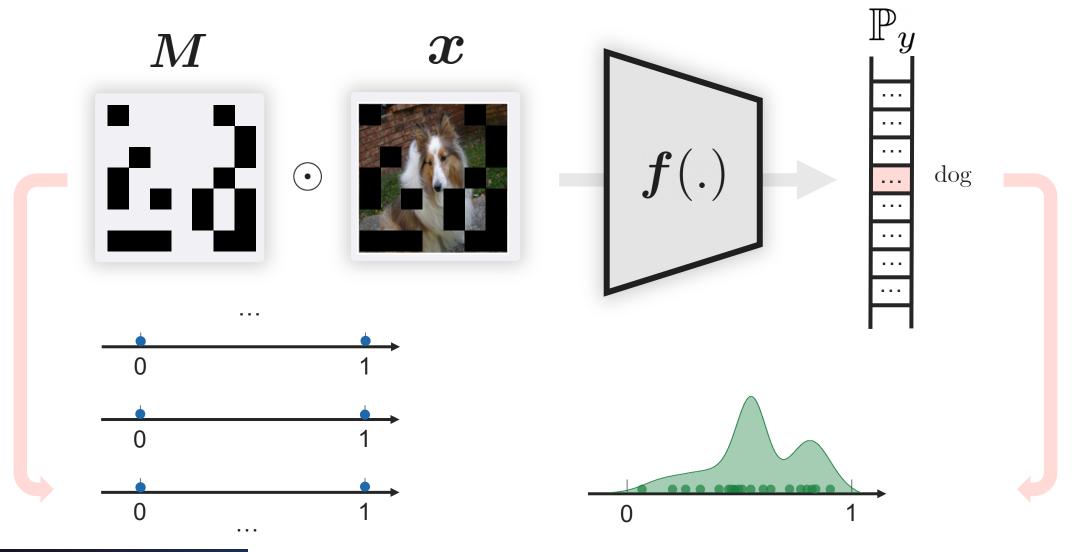






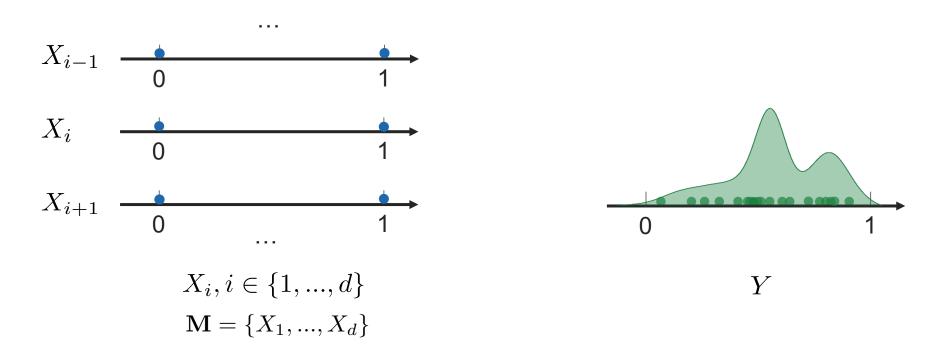






Black box: Patchwise Image Perturbation





How can we use these p samples?

Global sensitivity analysis



Let's consider a function
$$f: \begin{cases} \mathbb{R}^d & \to \mathbb{R} \\ X & \to Y = f(X) \end{cases}$$

Sensitivity analysis is concerned with measuring the **sensitivity** of Y to each **input vector** $X_i, i \in \{1, ..., d\}$. Here, $\mathbf{M} = \{X_1, ..., X_d\}$

Global sensitivity analysis is broadly used outside A.I.

- Classical statistics
- Industrial design optimization in engineering
- Physical modeling
- •

Global sensitivity analysis



Why Global?

GSA (as opposed to Local SA) considers the sensitivity of Y to X_i With respect to all its input domain.

Local

- Only considers the effect of X_i independently from one another
- Study the sensitivity of Y to small, local perturbations

Global

- Allows to draw general conclusions about the importance of a specific X_i
- Thorough analysis of the sensitivity of Y, including to interactions between X_i

Global sensitivity analysis



Some attribution methods perform SA without knowing it!

Local

Examples:

Occlusion



• Saliency

$$\Phi = \nabla f(x) \implies \phi_i = \frac{\partial f(x)}{\partial x_i}$$

Global

Examples:

• RISE

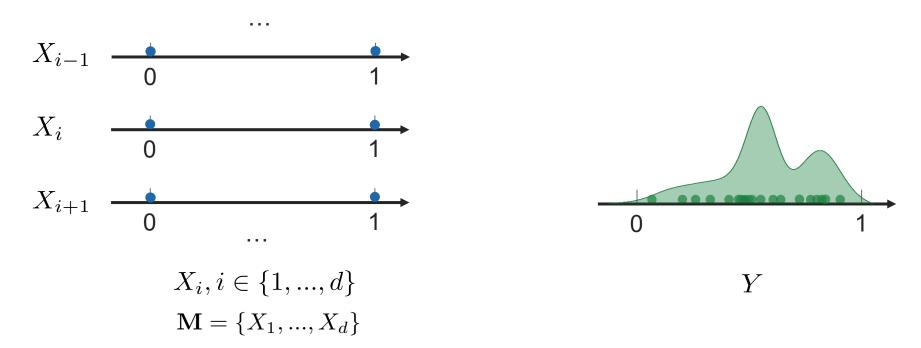


Sobol

• • •

Flashback: Sobol attribution method





Sobol method measures the importance of X_i by assessing its contribution to the variance of Y (ANOVA).

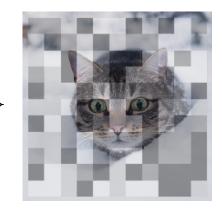
Flashback: Sobol attribution method [1]



Sobol method measures the importance of X_i by assessing its contribution to the variance of Y (ANOVA).

Each patch X_i gets an importance score which is the total Sobol index S_i of the corresponding X_i . Assesses the importance of X_i and of all its interactions.

$$\mathbf{M} = \{X_1, ..., X_d\}$$





 \mathcal{S}_i

[1] Thomas Fel et al, Neurips 2021

Aı

Another approach: GSA using dependence



Idea: if Y is sensitive to X_i , then those two random variables are dependent

How to measure the dependence between two random variables?

- Let \mathbb{P}_{X_i} be the probability distribution of X_i
- Let \mathbb{P}_Y be the probability distribution of Y
- Let $\|\cdot\|$ be some distance defined on probability distributions

$$\|\mathbb{P}_{X_i}\mathbb{P}_Y - \mathbb{P}_{X_i,Y}\| = 0 \Rightarrow \mathbb{P}_{X_i}\mathbb{P}_Y = \mathbb{P}_{X_i,Y} \Rightarrow X_i \perp Y$$

MMD and RKHS



One can measure the dependence between Y and X_i by assessing $\|\mathbb{P}_{X_i}\mathbb{P}_Y - \mathbb{P}_{X_i,Y}\|$

How to select $\|\cdot\|$? Two ingredients:

- Reproducing Kernel Hilbert Space (RKHS) is a space where we can construct representations (called embeddings) of random variables.
- The Maximum Mean Discrepancy (MMD) is a distance defined in a Restricted Kernel Hilbert Space (RKHS). It can be used to measure the distance between the embedding of two distributions.

MMD and RKHS



- Reproducing Kernel Hilbert Space (RKHS) is a space where we can construct representations (called embeddings) of random variables.
 - Let $k: \mathbb{R}^2 \to \mathbb{R}$ be a kernel
 - The embedding of $x \in \mathcal{X}$ In the RKHS \mathcal{F} , $\Phi : \mathcal{X} \to \mathcal{F}$ is defined by:

$$\Phi(x) := x' \to k(x, x')$$

• The Maximum Mean Discrepancy (MMD) is a distance defined in a Restricted Kernel Hilbert Space (RKHS). It can be used to measure the distance between the embedding of two distributions.

$$\gamma(P_{X_i}, P_Y) = MMD(P_{X_i}, P_Y) = \|\mu_{P_{X_i}} - \mu_{P_Y}\|_{\mathcal{H}}$$

Where $\mu_{P_{X_i}}$ is the mean embedding of X_i defined by $\mu_{P_{X_i}} := x' \to \int k(x, x') dP_{X_i}(x)$

Hilbert Schmidt Independence Criterion



- Let $k: \mathbb{R}^2 \to \mathbb{R}$ be a kernel used for embedding X_i , defining RKHS \mathcal{F}
- Let $l: \mathbb{R}^2 \to \mathbb{R}$ be a kernel used for embedding Y, defining RKHS \mathcal{G}
- Define a kernel $v: \mathbb{R}^4 \to \mathbb{R}^2$; $(x, x'), (y, y') \to k(x, x')l(y, y')$ and thus a RKHS \mathcal{H}

Hilbert Schmidt Independence Criterion is a measure of dependence defined on \mathcal{H} by

$$HSIC(X_i, Y) = \gamma^2(\mathbb{P}_{X_i}\mathbb{P}_Y, \mathbb{P}_{X_i, Y})$$

Hilbert Schmidt Independence Criterion



• Let $k: \mathbb{R}^2 \to \mathbb{R}$ be a kernel used for embedding X_i

• Let $l: \mathbb{R}^2 \to \mathbb{R}$ be a kernel used for embedding Y

(defines RKHSs \mathcal{G} and \mathcal{F})

HSIC can be efficiently estimated using:

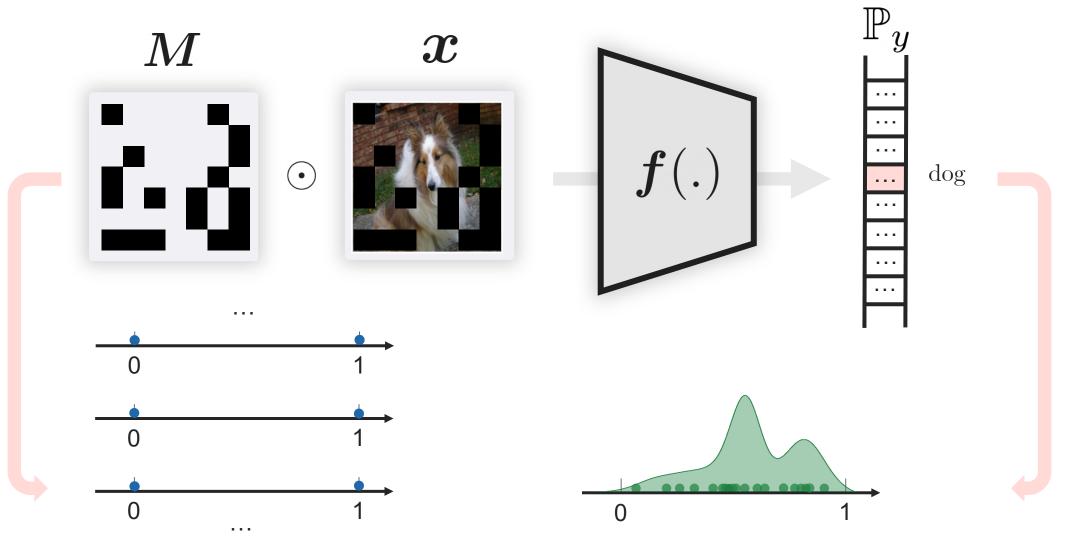
$$\mathcal{H}_{X_i,Y}^p = \frac{1}{(p-1)^2} \operatorname{tr}(KHLH)$$

where $H, L, K \in \mathbb{R}^{p \times p}$,

$$K_{jk} = k(X_i^j, X_i^k), L_{j,k} = l(Y^j, Y^k) \text{ and } H_{jk} = \delta(j = k) - p^{-1}$$

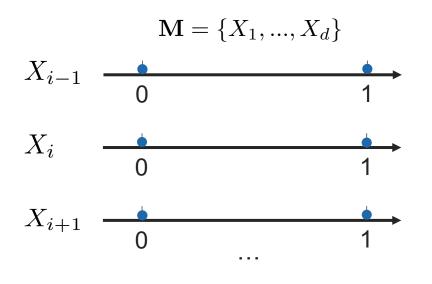
For an estimation with p samples $\{X_i^1,...,X_i^p\}$ of X_i

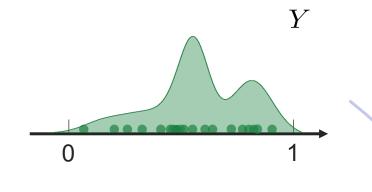


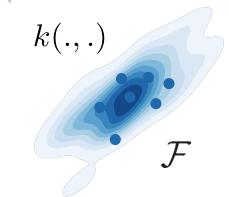


Hilbert Schmidt Independence Criterion









Hilbert Schmidt Independence Criterion:

$$HSIC(X_i, Y) = \gamma^2(\mathbb{P}_{X_i}\mathbb{P}_Y, \mathbb{P}_{X_i, Y})$$

Estimated with
$$\mathcal{H}^p_{X_i,Y} = \frac{1}{(p-1)^2} \operatorname{tr}(KHLH)$$

Advantages of HSIC



Why using a different sensitivity measure?

- o The estimator $\mathcal{H}^p_{X_i,Y}$ can estimate HSIC in $\mathcal{O}(1/\sqrt{p})$ with only p samples while Sobol estimator needs $p \times (d+2)$ samples to reach the same accuracy.
- o Bringing in RKHS theory opens up many research perspectives!

Practical Advantages: Efficiency



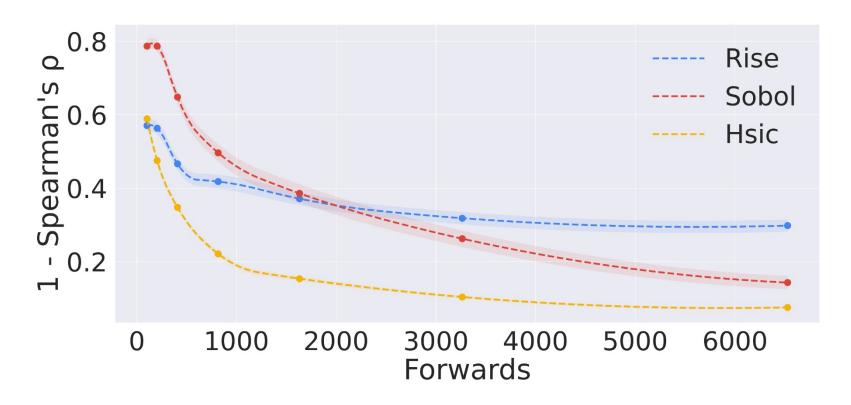
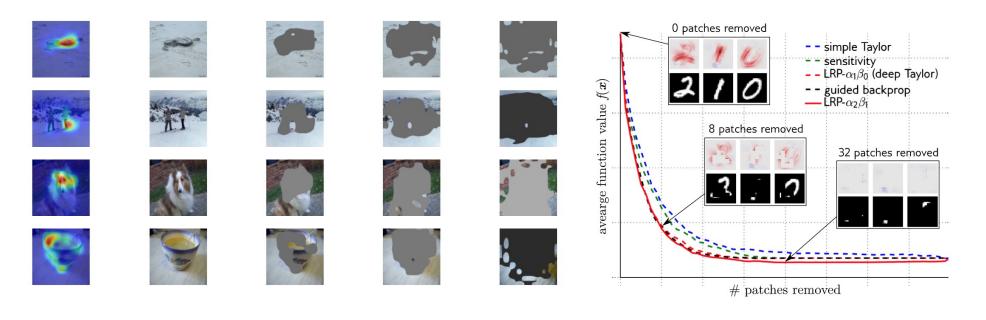


Illustration of the convergence speed of HSIC estimator against Sobol and RISE (Forwards = p)

How to evaluate the quality of explanations?



Fidelity metrics. Example: Deletion

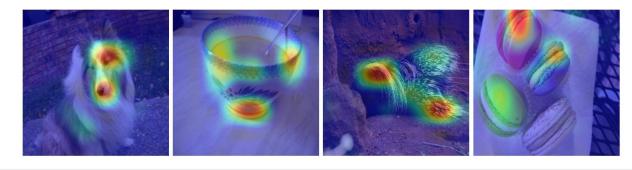


The better the explanation, the quicker the score should drop when removing important regions.

13/03/2023

First Results: Fidelity Metrics (Deletion)

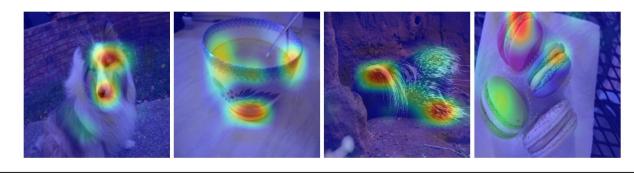




| | Method | ResNet50 | VGG16 | EfficientNet | MobileNetV2 |
|-----------|-------------------------------|----------|-------|---------------------|-------------|
| | | | | | |
| White-box | Saliency [43] | 0.158 | 0.120 | 0.091 | 0.113 |
| | GradInput [42] | 0.153 | 0.116 | 0.084 | 0.110 |
| | IntegGrad. [52] | 0.138 | 0.114 | 0.078 | 0.096 |
| | SmoothGrad [45] | 0.127 | 0.128 | 0.094 | 0.088 |
| | GradCAM++ [41] | 0.124 | 0.125 | 0.112 | 0.106 |
| | VarGrad [41] | 0.134 | 0.229 | 0.224 | 0.097 |
| Black-box | LIME [37] | 0.186 | 0.258 | 0.186 | 0.148 |
| | Kernel Shap [29] | 0.185 | 0.165 | 0.164 | 0.149 |
| | RISE [32] | 0.114 | 0.106 | 0.113 | 0.115 |
| | Sobol [11] | 0.121 | 0.109 | 0.104 | 0.107 |
| | \mathcal{H}_i^p eff. (ours) | 0.106 | 0.100 | 0.095 | 0.094 |
| | \mathcal{H}_i^p acc. (ours) | 0.105 | 0.099 | 0.094 | 0.093 |

First Results: Fidelity Metrics (Insertion)





| | Method | ResNet50 | VGG16 | <i>EfficientNet</i> | MobileNetV2 |
|-----------|-------------------------------|--------------|-------|---------------------|-------------|
| Ins. (†) | | | | | |
| White-box | Saliency [43] | 0.357 | 0.286 | 0.224 | 0.246 |
| | GradInput [42] | 0.363 | 0.272 | 0.220 | 0.231 |
| | IntegGrad. [52] | 0.386 | 0.276 | 0.248 | 0.258 |
| | SmoothGrad [45] | 0.379 | 0.229 | 0.172 | 0.246 |
| | GradCAM++ [41] | 0.497 | 0.413 | 0.316 | 0.387 |
| | VarGrad [41] | 0.527 | 0.241 | 0.222 | 0.399 |
| Black-box | LIME [37] | 0.472 | 0.273 | 0.223 | 0.384 |
| | Kernel Shap [29] | 0.480 | 0.393 | 0.367 | 0.383 |
| | RISE [32] | 0.554 | 0.485 | 0.439 | 0.443 |
| | Sobol [11] | 0.370 | 0.313 | 0.309 | 0.331 |
| | \mathcal{H}_i^p eff. (ours) | 0.470 | 0.387 | 0.357 | 0.381 |
| | \mathcal{H}_i^p acc. (ours) | <u>0.481</u> | 0.395 | <u>0.366</u> | 0.392 |

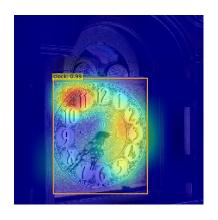
Practical Advantages: Efficiency

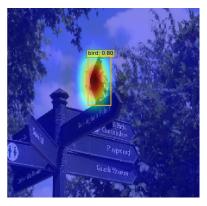


| · | | | | | | |
|-----------|-------------------------------|----------|-------|---------------------|-------------|----------------|
| | Method | ResNet50 | VGG16 | EfficientNet | MobileNetV2 | Exec. time (s) |
| | | | | | | |
| White-box | Saliency [43] | 0.158 | 0.120 | 0.091 | 0.113 | 0.360 |
| | GradInput [42] | 0.153 | 0.116 | 0.084 | 0.110 | 0.023 |
| | IntegGrad. [52] | 0.138 | 0.114 | 0.078 | 0.096 | 1.024 |
| | SmoothGrad [45] | 0.127 | 0.128 | 0.094 | 0.088 | 0.063 |
| | GradCAM++ [41] | 0.124 | 0.125 | 0.112 | 0.106 | 0.127 |
| | VarGrad [41] | 0.134 | 0.229 | 0.224 | 0.097 | 0.097 |
| Black-box | LIME [37] | 0.186 | 0.258 | 0.186 | 0.148 | 6.480 |
| | Kernel Shap [29] | 0.185 | 0.165 | 0.164 | 0.149 | 4.097 |
| | RISE [32] | 0.114 | 0.106 | 0.113 | 0.115 | 8.427 |
| | Sobol [11] | 0.121 | 0.109 | 0.104 | 0.107 | 5.254 |
| | \mathcal{H}_i^p eff. (ours) | 0.106 | 0.100 | 0.095 | 0.094 | 0.956 |
| | \mathcal{H}_i^p acc. (ours) | 0.105 | 0.099 | 0.094 | 0.093 | <u>1.668</u> |
| | | | | | | |

Explanations of Bounding Boxes











| Method | Deletion (\downarrow) | Insertion (↑) | μ Fidelity (\uparrow) | Exec. time (s) |
|--------------------------|-------------------------|---------------|-------------------------------|----------------|
| D-RISE [36] | 0.074 | 0.634 | 0.442 | 155 |
| Kernel Shap. [32] | 0.070 | 0.646 | 0.476 | 192 |
| \mathcal{H}_i^p (ours) | 0.088 | 0.658 | 0.568 | 34 |

Explanation of Yolov4 on COCO dataset

Shortcoming of HSIC: interactions



Let
$$A = \{l_1, ..., l_{|A|}\} \in \mathcal{P}_d$$
 i.e. a subset of $\{1, ..., d\}$

For **Sobol** indices, we have

$$S_A = \sum_{B \subset A} (-1)^{|A| - |B|} \frac{Var \mathbb{E}(Y|X_B)}{Var Y}$$

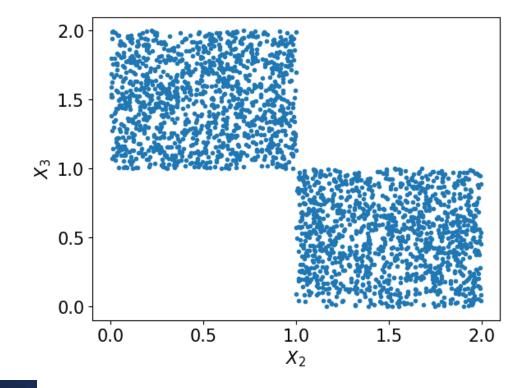
When $A = \{i, j\}$, $S_A = S_{i,j}$ can be simply obtained with

$$\mathcal{S}_{i,j} = egin{array}{c} ext{Not possible} \ ext{with HSIC} \end{array} egin{array}{c} \mathcal{S}_i - \mathcal{S}_j \ ext{(but expensive...)} \end{array}$$

Why considering interactions?



$$Y = f(X_1, X_2, X_3) = \begin{cases} 1 & \text{if } X_1 \in [0, 1], X_2 \in [1, 2], X_3 \in [0, 1], \\ 1 & \text{if } X_1 \in [0, 1], X_2 \in [0, 1], X_3 \in [1, 2], \\ 0 & \text{otherwise.} \end{cases}$$



Why considering interactions?



$$Y = f(X_1, X_2, X_3) = \begin{cases} 1 & \text{if } X_1 \in [0, 1], X_2 \in [1, 2], X_3 \in [0, 1], \\ 1 & \text{if } X_1 \in [0, 1], X_2 \in [0, 1], X_3 \in [1, 2], \\ 0 & \text{otherwise.} \end{cases}$$

- \circ X_1 is clearly important to explain Y
- \circ X_2 and X_3 are more difficult to interpret:

$$HSIC(\mathbf{x}_2, \mathbf{y}) = 0$$
 and $HSIC(\mathbf{x}_3, \mathbf{y}) = 0$

...whereas they clearly have an effect on. Y!

We have to look at interactions

ANOVA-like orthogonal decomposition of HSIC



In [1], an ANOVA like decomposition property is constructed for HSIC:

Let
$$A = \{l_1, ..., l_{|A|}\} \in \mathcal{P}_d$$
 i.e. a subset of $\{1, ..., d\}$

$$HSIC_A = \sum_{B \subset A} (-1)^{|A| - |B|} HSIC(X_B, Y)$$

When $A = \{i, j\}$, $HSIC_A = HSIC_{i,j}$ can be simply obtained with

$$HSIC_{i,j} = HSIC((X_i, X_j), Y) - HSIC(X_i, Y) - HSIC(X_j, Y)$$

...for a certain choice of kernel k_A

ANOVA-like orthogonal decomposition of HSIC



...for a certain choice of kernel k_A

$$k_{A}(X_{A}, X'_{A}) = \prod_{i \in A} (1 + k_{0}(X_{i}, X'_{i}))$$
with
$$k_{0}(X, X') = k(X, X') - \frac{\int k(X, t)dP(t) \int k(X', t)dP(t)}{\int \int k(s, t)dP(s)dP(t)}$$

Difficult to compute

Proposition: if the kernel is constructed as

$$k_{A}(X_{A}, X'_{A}) = \prod_{i \in A} (1 + k_{0}(X_{i}, X'_{i}))$$
with
$$k_{0}(X, X') = k(X, X') \frac{\int k(X, t) dP(t) \int k(X', t) dP(t)}{2 \int \int k(s, t) dP(s) dP(t)}$$

Interactions can be computed using orthogonal decomposition!

ANOVA-like orthogonal decomposition of HSIC



Proposition: if the kernel is constructed as

$$k_A(X_A, X_A') = \prod_{i \in A} (1 + k_0(X_i, X_i'))$$

with
$$k_0(X, X') = \delta(X = X') - \frac{1}{2}$$

Interactions can be computed using orthogonal decomposition!

Advantages of HSIC



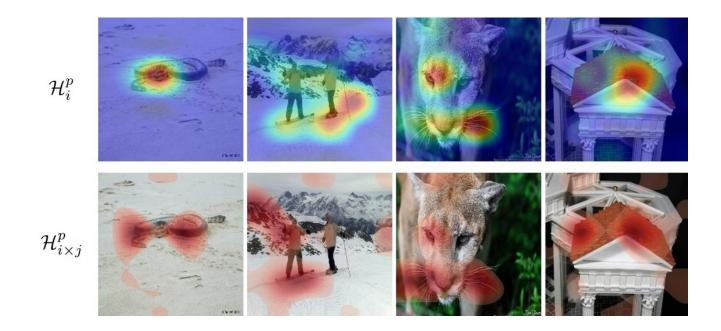
Why using a different sensitivity measure?

- The estimator $\mathcal{H}_{X_i,Y}^p$ can estimate HSIC in $\mathcal{O}(1/\sqrt{p})$ with only p samples while Sobol estimator needs $p \times (d+2)$ samples to reach the same accuracy.
- o Bringing in RKHS theory opens up many research perspectives!

Example: now, can assess pairwise interactions!

Practical Advantages: ANOVA decomposition





Conclusion and take away



Context

- Black box attribution methods based on patch perturbations are versatile and convenient ways of obtaining explanations
- They suffer from high computational costs because they need many forward passes
- Global sensitivity analysis is a promising approach to exploit these perturbation
- The current SOTA GSA based attribution method uses analysis of variance with Sobol indices.

We propose to use GSA based on dependence measures (HSIC)

- Needs less forward to obtain good explanations
- Theoretical advantages of RKHS
- Can assess patch-wise interactions