Postdoctoral position: Certified Stabilization of partial differential equations with prescribed solutions decay rate

1 Context

Hyperbolic partial differential equations provide a natural representation of industrial processes involving the evolution of quantities in time and space. In particular, hyperbolic partial differential equations provide a mathematical description of transport phenomena with finite speeds of propagation (e.g., transport of matter, sound waves, information). Related applications include wave propagation, electric transmission lines, hydraulic channels, drilling devices, or traffic flow [2, 4]. These systems are a source of complex control engineering problems and often have stringent environmental safety and economic feasibility constraints. Different theoretical approaches have been developed in the literature to tackle these control and observation questions. Among them, we can cite flatness-based controllers, optimization controllers, Lyapunov-based controllers [4], or recently the backstepping approach [14]. Although all of these approaches have enabled breakthroughs to control infinite-dimensional systems (with the design of explicit output-feedback laws), they suffer from several practical limitations. For instance, state-of-the-art feedback laws neglect the questions related to their complexity and numerical implementation. Moreover, only a few results have been obtained for the case of underactuated systems.

Interestingly, it has been shown in [3] that hyperbolic systems have equivalent stability properties as the ones of time-delay difference systems with distributed delays. Thus, it becomes possible to apply appropriate methods developed for time-delay systems to analyze quantitatively and qualitatively the closed-loop system properties or design appropriate and simple stabilizing controllers.

Recently, members of the L2S have set a new paradigm of Partial Poles Placement (PPP) for linear time-invariant functional differential equations and some classes of partial differential equations.

The PPP relies on two main strategies, themselves certified by the spectral distribution properties of time-delay systems: **multiplicity-induced-dominancy** (MID) and **coexisting-real-roots-induced-dominancy** (CRRID). Exploring some previous ideas present, e.g., in [19], the seminal works [9, 12] highlighted the fact that spectral values attaining their maximal admissible multiplicity tend to be dominant, in what came to be known as the MID property. Since these first works, several research papers, such as [11, 7, 8, 18], have addressed theoretical and applied questions on the MID property, aiming at understanding for which classes of time-delay systems such a link between the root of maximal multiplicity and rightmost root exists. Other recent works have also explored links between roots with high multiplicity, but not necessarily maximal, and dominancy [10, 16, 18]. When available, this property can be helpful in the stabilization of time-delay systems since it suffices to select the system's free parameters in order to guarantee the existence of such a root of maximal multiplicity with negative real part, and the MID property will ensure its dominance. Several recent works, such as [11, 8, 18], have considered applications of the MID property in the stabilization of **time-delay systems**.

Instead of assigning a single root of maximal multiplicity, some recent works such as [5] have considered the assignment of as many simple real roots as the maximal possible multiplicity for a root and shown that, in several situations, the rightmost root of those assigned is dominant (for the whole spectrum), consisting in what has been named the CRRID property. Assigning several simple real roots instead of a single root of large multiplicity allows for weaker constraints in feedback control design, which has been explored in applications in [7, 8]. The main ingredient of the described results is an integral representation of the corresponding quasipolynomial in both MID and CRRID properties. In the case of the MID, such an integral representation appears to be nothing but the well-known Kummer hypergeometric function, see for instance [17]. However, in the case of the CRRID, generalized hypergeometric functions are involved. It turns out that such special functions are equally valuable tools for time-domain stability analysis of time-delay systems, see for instance [15].

Consequently, the MID and CRRID properties could be used to stabilize hyperbolic systems. This would pave the way for a new generation of stabilizing controllers that are simple to implement (and consequently do not require an expensive computational cost) while explicitly taking into account the delays and high-frequency content in the model (which should lead to overall increased performance).

2 Work program

Despite the numerous works and finding on this topic, several theoretical questions remain open.

1. The MID and the CRRID properties were studied essentially only for systems with a single delay (except for first-order systems in [13]), and the effective techniques developed by members of the team in this direction cannot be immediately generalized to the multi-delay case or the case of distributed delays. Thus, an important goal for the project's sequel is to study such dominancy properties in the multi-delay and distributed-delay contexts and extend such a property to classes of hyperbolic partial differential equations.

2. We will also investigate the case of underactuated hyperbolic systems since they can be rewritten as time-delay systems with distributed delayed actuation [20]. Designing stabilizing controllers can be a difficult task for such problems, and the MID/CRRID properties could be a significant asset.

3. Finally, we aim to compare the proposed control strategies with state-of-the-art controllers (such as backstepping or flatness-based controllers) with respect to a set of specifications and performance criteria (sensitivity, robustness margins, control effort, data sampling, convergence rate, and computing power). Such work has been initiated in [1] for comparing different control strategies to eliminate stick-slip during drilling operations. The MID controllers presented satisfying performance (although not as good as complex control strategies) associated with a low computational cost (which was not the case of the complex strategies).

A promising research direction towards this goal is to exploit connections between the spectra of time-delay systems and roots of hypergeometric functions, as highlighted in [17, 6] (in the case of the MID property) for the single-delay case. New results concerning partial pole placement for time-delay systems are also planned to be integrated into the P3δ software [7, 8].

3 Contact

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References

[1] Jean Auriol, Islam Boussaada, Roman J Shor, Hugues Mounier, and Silviu-Iulian Niculescu. Comparing advanced control strategies to eliminate stick-slip oscillations in drillstrings. *IEEE Access*, 10:10949–10969, 2022.

[2] Jean Auriol, Joachim Deutscher, Guilherme Mazanti, and Giorgio Valmorbida. *Advances in Distributed Parameter Systems*. Springer, 2022.

[3] Jean Auriol and Florent Di Meglio. An explicit mapping from linear first order hyperbolic pdes to difference systems. *Systems & Control Letters*, 123:144–150, 2019.

[4] Georges Bastin and Jean-Michel Coron. *Stability and boundary stabilization of 1-d hyperbolic systems*, volume 88. Springer, 2016.

[5] Fazia Bedouhene, Islam Boussaada, and Silviu-Iulian Niculescu. Real spectral values coexistence and their effect on the stability of time-delay systems: Vandermonde matrices and exponential decay. *Comptes Rendus. Mathématique*, 358(9-10):1011–1032, September 2020.

[6] Islam Boussaada, Guilherme Mazanti, and Silviu-Iulian Niculescu. The generic multiplicity-induced-dominancy property from retarded to neutral delay-differential equations: When delay-systems characteristics meet the zeros of Kummer functions. *Comptes Rendus. Mathématique*, 2022.

[7] Islam Boussaada, Guilherme Mazanti, Silviu-Iulian Niculescu, Julien Huynh, Franck Sim, and Matthieu Thomas. Partial Pole Placement via Delay Action: A Python Software for Delayed Feedback Stabilizing Design. In *ICSTCC 2020 - 24th International Conference on System Theory, Control and Computing*, 2020 24th International Conference on System Theory, Control and Computing (ICSTCC), pages 196–201, Sinaia, Romania, October 2020. `https://arxiv.org/abs/2007.05809`.

[8] Islam Boussaada, Guilherme Mazanti, Silviu-Iulian Niculescu, Adrien Leclerc, Jayvir Raj, and Max Perraudin. New Features of P3δ software: Partial Pole Placement via Delay Action. *IFAC-PapersOnLine*, 54(18):215–221, September 2021. TDS 2021 - 16th IFAC Workshop on Time Delay Systems.

[9] Islam Boussaada and Silviu-Iulian Niculescu. Characterizing the codimension of zero singularities for time-delay systems: a link with Vandermonde and Birkhoff incidence matrices. *Acta Appl. Math.*, 145:47–88, 2016.

[10] Islam Boussaada, Silviu-Iulian Niculescu, Ali El Ati, Redamy Pérez-Ramos, and Karim Liviu Trabelsi. Multiplicity-Induced-Dominancy in parametric second-order delay differential equations: Analysis and application in control design. *ESAIM: Control, Optimisation and Calculus of Variations*, November 2019.

[11] Islam Boussaada, Sami Tliba, Silviu-Iulian Niculescu, Hakki Ulaş Unal, and Tomáš Vyhlídal. Further remarks on the effect of multiple spectral values on the dynamics of time-delay systems. Application to the control of a mechanical system. *Linear Algebra and its Applications*, 542:589–604, April 2018.

[12] Islam Boussaada, Hakki Ulaş Unal, and Silviu-Iulian Niculescu. Multiplicity and stable varieties of time-delay systems: A missing link. In *22nd International Symposium on Mathematical Theory of Networks and Systems (MTNS)*, 2016.

[13] Sébastien Fueyo, Guilherme Mazanti, Islam Boussaada, Yacine Chitour, and Silviu-Iulian Niculescu. Insights into the multiplicity-induced-dominancy for scalar delay-differential equations with two delays. *IFAC-PapersOnLine*, 54(18):108–114, September 2021. TDS 2021 - 16th IFAC Workshop on Time Delay Systems.

[14] Miroslav Krstic and Andrey Smyshlyaev. *Boundary control of PDEs: A course on backstepping designs.* SIAM, 2008.

[15] Kun Liu, Alexandre Seuret, Yuanqing Xia, Frédéric Gouaisbaut, and Yassine Ariba. Bessel–Laguerre inequality and its application to systems with infinite distributed delays. *Automatica*, 109:108562, November 2019.

[16] Dan Ma, Islam Boussaada, Jianqi Chen, Catherine Bonnet, Silviu-Iulian Niculescu, and Jie Chen. PID Control Design for First-Order Delay Systems via MID Pole Placement: Performance vs. Robustness. *Automatica*, December 2021.

[17] Guilherme Mazanti, Islam Boussaada, and Silviu-Iulian Niculescu. Multiplicity-induced-dominancy for delay-differential equations of retarded type. *Journal of Differential Equations*, 286(15):84–118, March 2021.

[18] Silviu-Iulian Niculescu, Islam Boussaada, Xu-Guang Li, Guilherme Mazanti, and César-Fernando Méndez-Barrios. Stability, Delays and Multiple Characteristic Roots in Dynamical Systems: A Guided Tour. *IFAC-PapersOnLine*, 54(18):222–239, September 2021. 16th IFAC Workshop on Time Delay Systems (IFAC TDS 2021).

[19] Edmund Pinney. *Ordinary difference-differential equations.* University of California Press, Berkeley-Los Angeles, 1958.

[20] Jeanne Redaud, Jean Auriol, and Silviu-Iulian Niculescu. Stabilizing output-feedback control law for hyperbolic systems using a fredholm transformation. *IEEE Transactions on Automatic Control*, 67(12):6651–6666, 2022.