Massive MIMO Radar for Target Detection

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Outline

- **Motivation**: a robust target detection problem.
- **Main goals**:
  1. Derive a detector whose asymptotic distribution is invariant with respect to the unknown distribution of the disturbance.
  2. Maximize the probability of detection (P_D) while keeping a constant probability of false alarm (P_{FA}).
- **How to achieve it**:
  1. Increase the spatial degrees of freedom (DoF) using a co-colocated MIMO radar.[1,2]
  2. Robust and misspecified statistics (constant P_{FA}).[1]
  3. Reinforcement Learning (P_D maximization).[2]

[1] S. Fortunati, L. Sanguinetti, F. Gini, M. S. Greco and B. Himed, "Massive MIMO Radar for Target Detection", in *IEEE Transactions on Signal Processing*, vol. 68, pp. 859-871, 2020.

[2] A. M. Ahmed, A. A. Ahmad, S. Fortunati, A. Sezgin, M. S. Greco and F. Gini, "A Reinforcement Learning Based Approach for Multitarget Detection in Massive MIMO Radar," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 57, no. 5, pp. 2622-2636, Oct. 2021.

The target detection problem

- Consider a multiple antenna radar system with N spatial channels, collecting K temporal snapshots $\{\mathbf{x}_k\}_{k=1}^{K} \in \mathbb{C}^N$.

- **Detection problem**:

$$\begin{array}{lll} H_0: & \mathbf{x}_k = \mathbf{c}_k & k = 1, \ldots, K, \\ H_1: & \mathbf{x}_k = \alpha_k \mathbf{v}_k + \mathbf{c}_k & k = 1, \ldots, K, \end{array}$$

  - $\mathbf{v}_k \in \mathbb{C}^N$: known at each time instant $k \in \{1, \ldots, K\}$,
  - $\alpha_k \in \mathbb{C}$: deterministic, *unknown*, scalar that may vary over k,
  - $\mathbf{C} \triangleq [\mathbf{c}_1, \ldots, \mathbf{c}_k]$: disturbance.

- A decision statistic $\Lambda(\mathbf{X})$ needs to be implemented:

$$\Lambda(\mathbf{X}) \underset{H_0}{\overset{H_1}{\gtrless}} \lambda, \qquad \mathbf{X} \triangleq [\mathbf{x}_1, \ldots, \mathbf{x}_K].$$

How to choose the threshold

- The threshold λ should be chosen to maintain the P_{FA} below a pre-assigned value:

$$\Pr\{\Lambda(\mathbf{X}) > \lambda | H_0\} = \int_{\lambda}^{\infty} p_{\Lambda|H_0}(a|H_0)da = \overline{P_{FA}}.$$

- $p_{\Lambda|H_0}$ is the pdf of $\Lambda(\mathbf{X})$ under the null hypothesis H_0.

- Three simplifying assumptions are generally adopted:
  - M1 $\{\mathbf{c}_k\}_{k=1}^{K}$ are i.i.d. over the observation interval,
  - M2 α_k maintains constant over k: $\alpha_k \equiv \alpha$, $\forall k$,
  - M3 The pdf $p_{\mathbf{C}}(\mathbf{C}) = \prod_{k=1}^{K} p_C(\mathbf{c}_k)$ is perfectly known.

Perfectly matched GLR

- Under M1, M2 and M3, the Generalized Likelihood Ratio (GLR) statistic $\Lambda_{\mathrm{GLR}}(\mathbf{X})$ can be derived.

- Under H_0, as the number of temporal snapshots grows to infinity ($K \to \infty$), we get:[3]

$$\Lambda_{\mathrm{GLR}}(\mathbf{X}|H_0) \underset{K\to\infty}{\sim} \chi_2^2(0).$$

- Consequently, an asymptotic solution for λ is: $\bar{\lambda} = -2\ln \overline{P_{FA}}$.

Is it possible to derive a detection statistic with the same asymptotic properties of $\Lambda_{\mathrm{GLR}}(\mathbf{X})$ without relying on Assumptions M1, M2 and M3?

[3] S. M. Kay, *Fundamentals of statistical signal processing, volume II: detection theory*, Prentice Hall, 1993.

Spatial asymptotic regime

- We collect a single temporal snapshot ($K = 1$) and exploit the spatial dimension N:

$$\begin{aligned} H_0 &: \quad \mathbf{x} = \mathbf{c} \\ H_1 &: \quad \mathbf{x} = \alpha \mathbf{v} + \mathbf{c}, \end{aligned}$$

- This allows us to entirely drop Assumptions M1 and M2.

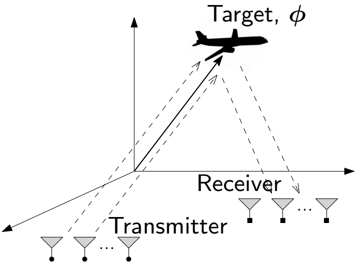
Note that, unlike in the temporal domain, the spatial samples $x_1, \ldots, x_N$ *cannot* be considered as *independent* observations!

- We use advances in robust and misspecified statistics [4] in the presence of *dependent data* to dispose of M3.

[4] H. White and I. Domowitz, "Nonlinear regression with dependent observations," *Econometrica*, vol. 52, no. 1, pp. 143–161, 1984.

Co-located MIMO system model

- We need radar systems with a large number N of spatial DoF: **co-located MIMO radars**



- M_T transmitting antennas,
- M_R receiving antennas,
- $N \triangleq M_T M_R$: *virtual spatial antenna channels*.

- Signal collected at the receiving array:

$$\mathbf{x}(t) = \bar{\alpha}\mathbf{a}_R(\bar{\phi})\mathbf{a}_T^T(\bar{\phi})\mathbf{s}(t-\bar{\tau})e^{j\bar{\omega}t} + \mathbf{n}(t), \ t \in [0, T]$$

- $\mathbf{a}_T(\phi) \in \mathbb{C}^{M_T}$: transmitting steering vector,
- $\mathbf{a}_R(\phi) \in \mathbb{C}^{M_R}$: receiving steering vector.

Continuous-time signal model

$$\mathbf{x}(t) = \bar{\alpha}\mathbf{a}_R(\bar{\phi})\mathbf{a}_T^T(\bar{\phi})\mathbf{s}(t-\bar{\tau})e^{j\bar{\omega}t} + \mathbf{n}(t),\ t \in [0, T]$$

- $\mathbf{x}(t) \in \mathbb{C}^{M_R}$: array output vector at time t,
- $\mathbb{C}^{M_T} \ni \mathbf{s}(t) \triangleq \mathbf{W}\mathbf{s}_o(t)$: vector of transmitted signals
  - $\mathbf{W} \in \mathbb{C}^{M_T \times M_T}$ is the waveforms weighting matrix,
  - $\mathbf{s}_o(t)$: vector of *nearly* orthonormal signals,
- $\mathbf{n}(t) \in \mathbb{C}^{M_R}$: complex disturbance random process, or *clutter*.
- $\bar{\alpha} \in \mathbb{C}$ accounts for target RCS and two-way path losses.

Co-located MIMO radar

$\bar{\alpha}$ is the same for each transmitter and receiver pair.

Discrete-time signal model (1/2)

- The output matrix $\mathbf{X}(l, k)$ of the filter matched to $\mathbf{s}_o(t)$ is:[5]

$$\mathbb{C}^{M_R \times M_T} \ni \mathbf{X}(l, k) = \bar{\alpha}\mathbf{a}_R(\bar{\phi})\mathbf{a}_T(\bar{\phi})^T\mathbf{W}\mathbf{S}(l, k) + \mathbf{C}(l, k),$$

- "Straddling loss" matrix:

$$\mathbf{S}(l, k) \triangleq \int_0^T \mathbf{s}_o(t - \bar{\tau})\mathbf{s}_o^H(t - l\Delta t)e^{-j(k\Delta\omega - \bar{\omega})t}dt$$

- Disturbance matrix:

$$\mathbf{C}(l, k) \triangleq \int_0^T \mathbf{n}(t)\mathbf{s}_o^H(t - l\Delta t)e^{-jk\Delta\omega t}dt.$$

- The range-Doppler indices (l, k) will be omitted next.

[5] B. Friedlander, "On signal models for MIMO radar," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 48, no. 4, pp. 3655–3660, October 2012.

Discrete-time signal model (1/2)

- The output matrix $\mathbf{X}$ can be expressed as:

$$\mathbb{C}^N \ni \mathbf{x} = \mathrm{vec}\left(\mathbf{X}\right) = \bar{\alpha}\mathbf{v}(\bar{\phi}) + \mathbf{c}$$

where $\mathbf{c} \triangleq \mathrm{vec}\left(\mathbf{C}\right)$ and:

$$\mathbf{v}(\bar{\phi}) = (\mathbf{S}^T \otimes \mathbf{I}_{M_R}) \left[\mathbf{W}^T \mathbf{a}_T(\bar{\phi}) \otimes \mathbf{a}_R(\bar{\phi})\right].$$

- If $\mathbf{n}(t)$ is a wide-sense stationary process, we have:

$$E\{\mathbf{n}(t)\} = \mathbf{0}, \forall t \quad \Rightarrow \quad E\{\mathbf{c}\} = \mathbf{0}$$

$$E\{\mathbf{n}(t)\mathbf{n}(\tau)^H\} = \mathbf{\Sigma}(t-\tau) \quad \Rightarrow \quad \mathbf{\Gamma} \triangleq E\{\mathbf{c}\mathbf{c}^H\}$$

$$\mathbf{\Gamma} = \iint \left[\mathbf{s}_o^*(t - l\Delta t)\mathbf{s}_o^T(t - l\Delta t) \otimes \mathbf{\Sigma}(t-\tau)\right] e^{-jk\Delta\omega(t-\tau)} dt d\tau.$$

Fully uncorrelated disturbance model

- Assumptions on the clutter process $\mathbf{n}(t)$:
  1. $\mathbf{n}(t)$ is *spatially uncorrelated* (along the receiving array),
  2. $\mathbf{n}(t)$ is also *temporally uncorrelated* (along T),

$$E\{\mathbf{n}(t)\mathbf{n}(\tau)^H\} = \mathbf{\Sigma}(t-\tau) = \sigma^2 \mathbf{I}_{M_R}\delta(t-\tau).$$

- If *perfect orthogonality* of the waveforms in $\mathbf{s}_o(t)$ is assumed:

$$\mathbf{\Gamma} = \sigma^2 \mathbf{I}_N = \begin{pmatrix} \sigma^2 & 0 & \cdots & 0 \\ 0 & \sigma^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma^2 \end{pmatrix}.$$

- This is a simple but **very unrealistic** model!

Temporally uncorrelated disturbance model

- Assumption on the clutter process $\mathbf{n}(t)$:
  1. $\mathbf{n}(t)$ is *temporally uncorrelated* (along T),

$$E\{\mathbf{n}(t)\mathbf{n}(\tau)^H\} = \mathbf{\Sigma}(t-\tau) = \mathbf{\Sigma}_R \delta(t-\tau).$$

- If *perfect orthogonality* of the waveforms in $\mathbf{s}_o(t)$ is assumed:

$$\mathbf{\Gamma} = \mathbf{I}_{M_T} \otimes \mathbf{\Sigma}_R = \begin{pmatrix} \mathbf{\Sigma}_R & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{\Sigma}_R & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{\Sigma}_R \end{pmatrix}.$$

- This is a more complex but **still unrealistic** model!

A more general disturbance models (1/2)

- We drop the too stringent assumptions on
  - the *temporal uncorrelation* of $\mathbf{n}(t)$,
  - the *perfect orthogonality* of $\mathbf{s}_o(t)$,

  in favour of a much weaker requirement.

Our assumption

$[\mathbf{\Gamma}]_{i,j}$ goes to zero at least polynomially fast as $|i-j|$ increases.[6]

- Moreover, unlike most of the existing literature, we **do not** require $\mathbf{c}$ to be *Gaussian-distributed*.

[6] S. Fortunati, L. Sanguinetti, F. Gini, M. S. Greco and B. Himed, "Massive MIMO Radar for Target Detection", in *IEEE Transactions on Signal Processing*, vol. 68, pp. 859-871, 2020.

A more general disturbance models (2/2)

- More formally, let $\mathbf{c} = [c_1, \ldots, c_N]^T$ be the disturbance vector.
- The entries $\{c_n\}_{n=1}^N$ can be considered as random variables sampled form a stationary discrete-time process $\{c_n : \forall n\}$.

Assumption A1: The autocorrelation function (ACF) of $\{c_n : \forall n\}$ satisfies

$$r_C[m] \triangleq E\{c_n c_{n-m}^*\} = O(|m|^{-\gamma})$$

where $m \in \mathbb{Z}$, $\gamma > \varrho/(\varrho - 1)$, $\varrho > 1$. [7]

- Note that we are not assuming any particular pdf p_C for $\mathbf{c}$, that will be left unspecified!

[7] H. White and I. Domowitz, "Nonlinear regression with dependent observations," *Econometrica*, vol. 52, no. 1, pp. 143–161, 1984.

Example 1: ARMA model

- A stable $\mathrm{ARMA}(p, q)$ process, with finite p and q, satisfies Assumption A1 since its ACF decays exponentially fast.
- The second-order statistics of any discrete-time process with *continuous* Power Spectra Density (PSD) can be well-approximated by an ARMA model[8].
- A subset of the general ARMA models are the autoregressive model of order p, $\mathrm{AR}(p)$.
- AR models share most of the properties of the ARMA models.

[8] J. Li and P. Stoica, *MIMO Radar Signal Processing*. Hoboken, NJ: Wiley, 2009.

Example 2: AR model

- A stable *stationary* AR(p) process $\{c_n : \forall n\}$ is a discrete random process s.t.:

$$c_n = \sum_{i=1}^{p} \rho_i c_{n-i} + w_n, \quad n \in (-\infty, \infty).$$

- The innovations w_n are zero-mean, *circularly symmetric*, i.i.d. random variables with $E\{|w_n|^2\} = \sigma_w^2 < \infty$.

- The pdf of w_n, say $p_W(w; \boldsymbol{\varphi})$ is generally non-Gaussian and may depends on an additional unknown *nuisance* vector $\boldsymbol{\varphi}$.

- The ACF of a AR(p) process decays exponentially fast, so its satisfy Assumption A1.

Example 3: Compound Gaussian (CG) model

- Any CG-distributed vector $\mathbf{c}$ admits a representation:

$$\mathbf{c} =_d \sqrt{\tau}\mathbf{z},$$

where:

  - the *texture* τ is a positive random variable,
  - the *speckle* $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Gamma})$ is a complex Gaussian random vector with scatter/covariance matrix $\boldsymbol{\Gamma}$.

- The entries $\{z_n\}_{n=1}^N$ of the speckle can be considered as samples of a Gaussian ARMA(p, q) $\{z_n : \forall n\}$ with ACF $r_Z[m]$.

- The speckle scatter matrix is then given by $[\boldsymbol{\Gamma}]_{i,j} = r_Z[i-j]$, with $1 \leq i, j \leq N$.

- It is immediate to verify that the CG model satisfy A1.

A robust HT problem

- Let us recall the MIMO detection problem:

$$\begin{aligned} H_0 : &\quad \mathbf{x} = \mathbf{c} \\ H_1 : &\quad \mathbf{x} = \bar{\alpha}\mathbf{v} + \mathbf{c}, \end{aligned}$$

where:

- $\mathbf{v} \equiv \mathbf{v}(\phi) = (\mathbf{S}^T \otimes \mathbf{I}_{M_R}) \left[\mathbf{W}^T \mathbf{a}_T(\phi) \otimes \mathbf{a}_R(\phi)\right]$ is a known steering vector,
- $\bar{\alpha}$ is a deterministic unknown,
- $\mathbf{c}$ is the disturbance vector that is assumed to satisfy Assumption A1 but whose pdf p_C is *unknown*.

Final goal

Find a robust decision statistic whose asymptotic (as $N \to \infty$) distribution under H_0 does not depend on the *unknown* disturbance pdf p_C.

Main results: estimation

- The Least Square (LS) estimator of $\bar{\alpha}$ is $\hat{\alpha} = \mathbf{v}^H \mathbf{x} / ||\mathbf{v}||^2$.

Theorem 1

Under Assumption A1, the LS estimator $\hat{\alpha}$ is: [9,10]

1. *Consistent*: $\hat{\alpha} \underset{N \to \infty}{\overset{p}{\to}} \bar{\alpha}$,

2. *Asymptotically normal*: $\sqrt{N} \bar{B}_N^{-1/2} A_N (\hat{\alpha} - \bar{\alpha}) \underset{N \to \infty}{\sim} \mathcal{CN}(0,1)$,

$$A_N \triangleq N^{-1} ||\mathbf{v}||^2, \quad \bar{B}_N \triangleq N^{-1} \mathbf{v}^H \boldsymbol{\Gamma} \mathbf{v}, \quad \boldsymbol{\Gamma} \triangleq E_{p_C}\{\mathbf{c}\mathbf{c}^H\},$$

with p_C being the unknown disturbance pdf.

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[9] H. White and I. Domowitz, "Nonlinear regression with dependent observations," *Econometrica*, vol. 52, no. 1, pp. 143–161, 1984.

[10] S. Fortunati, L. Sanguinetti, F. Gini, M. S. Greco and B. Himed, "Massive MIMO Radar for Target Detection", in *IEEE Transactions on Signal Processing*, vol. 68, pp. 859-871, 2020.

A consistent estimator for $\bar{B}_N$ (1/2)

- The scalar $\bar{B}_N$ is function of the unknown disturbance covariance matrix $\mathbf{\Gamma}$.

- A consistent estimator $\hat{B}_N$ of $\bar{B}_N$ is:

$$\hat{B}_N \equiv \hat{B}_N(\hat{\alpha}) = N^{-1}\mathbf{v}^H\widehat{\mathbf{\Gamma}}_l\mathbf{v},$$

where

$$[\widehat{\mathbf{\Gamma}}_l]_{i,j} \triangleq \begin{cases} \hat{c}_i\hat{c}_j^* & 0 \leq j-i \leq l \\ \hat{c}_i^*\hat{c}_j & 0 \leq i-j \leq l \\ 0 & |i-j| > l \end{cases} \quad 1 \leq i,j \leq N,$$

$$\hat{c}_n = x_n - \hat{\alpha}v_n,\ \forall n \quad \hat{\alpha} = \mathbf{v}^H\mathbf{x}/||\mathbf{v}||^2,$$

and l is the so-called *truncation lag*.

A consistent estimator for $\bar{B}_N$ (2/2)

Theorem 2

Under Assumption A1, if $l \to \infty$ as $N \to \infty$ such that $l = o(N^{1/3})$ then: [11]

$$\hat{B}_N - \bar{B}_N \underset{N\to\infty}{\overset{p}{\to}} 0.$$

- Theorems 1 and 2 tell us that, irrespective of the unknown p_C, the LS estimator $\hat{\alpha}$ is:
  - $\sqrt{N}$-consistent,
  - asymptotically normal estimator with asymptotic error covariance matrix given by $A_N^{-1}\bar{B}_N$,
  - a consistent estimate of $\bar{B}_N$ is provided by $\hat{B}_N$.
- A **Wald-type test** can be implemented!

[11] H. White and I. Domowitz, "Nonlinear regression with dependent observations," *Econometrica*, vol. 52, no. 1, pp. 143–161, 1984.

Main results: detection

A robust Wald-type test

The asymptotic characterization of the LS estimator leads to: [12]

$$\Lambda_{\mathrm{RW}}(\mathbf{x}) = \frac{2N|\hat{\alpha}|^2}{A_N^{-2}\hat{B}_N} = \frac{2|\mathbf{v}^H\mathbf{x}|^2}{\mathbf{v}^H\widehat{\boldsymbol{\Gamma}}_l\mathbf{v}}.$$

Theorem 3

If Assumption A1 holds true, then:

$$\Lambda_{\mathrm{RW}}(\mathbf{x}|H_0) \underset{N\to\infty}{\sim} \chi_2^2(0),$$
$$\Lambda_{\mathrm{RW}}(\mathbf{x}|H_1) \underset{N\to\infty}{\sim} \chi_2^2(\varsigma),$$

where $\varsigma \triangleq 2|\bar{\alpha}|^2 \frac{||\mathbf{v}||^4}{\mathbf{v}^H\boldsymbol{\Gamma}\mathbf{v}}$.

[12] S. Fortunati, L. Sanguinetti, F. Gini, M. S. Greco and B. Himed, "Massive MIMO Radar for Target Detection", in *IEEE Transactions on Signal Processing*, vol. 68, pp. 859-871, 2020.

On the non-centrality parameter ς

- An explicit expression for ς is given by:

$$\varsigma = \frac{2|\bar{\alpha}|^2 M_R^2 \|(\mathbf{WS})^T \mathbf{a}_T(\bar{\phi})\|^4}{\mathrm{tr}\left(\boldsymbol{\Gamma}\left[(\mathbf{WS})^T \mathbf{a}_T(\bar{\phi})\mathbf{a}_T^H(\bar{\phi})(\mathbf{WS})^* \otimes \mathbf{a}_R(\bar{\phi})\mathbf{a}_R^H(\bar{\phi})\right]\right)}.$$

- By substituting $\boldsymbol{\Gamma}$ with its definition, we get:

$$\varsigma = \frac{2|\bar{\alpha}|^2 M_R \|(\mathbf{WS})^T \mathbf{a}_T(\bar{\phi})\|^2}{\iint_0^T ||\mathbf{s}_o(t - \bar{l}\Delta t)||^2 \mathrm{tr}\left[\boldsymbol{\Sigma}(t-\tau)\right] e^{-j\bar{k}\Delta\omega(t-\tau)} dt d\tau}.$$

- If $\mathbf{S} = \mathbf{I}_{M_T}$ and $\boldsymbol{\Sigma}(t-\tau) = \sigma^2 \mathbf{I}_{M_R}\delta(t-\tau)$: [13]

$$\varsigma = \frac{2|\bar{\alpha}|^2 P(\bar{\phi})}{\sigma^2}, \quad P(\bar{\phi}) \triangleq \mathbf{a}_T^H(\bar{\phi})\mathbf{W}^*\mathbf{W}^T\mathbf{a}_T(\bar{\phi}),$$

where $P(\bar{\phi})$ is the transmitting beam pattern.

[13] I. Bekkerman and J. Tabrikian, "Target detection and localization using MIMO radars and sonars," *IEEE Transactions on Signal Processing*, vol. 54, no. 10, pp. 3873–3883, Oct 2006

Asymptotic performance of $\Lambda_{\mathrm{RW}}(\mathbf{x})$

CFAR property and ROC curve

Under A1, the P_{FA} of $\Lambda_{\mathrm{RW}}(\mathbf{x})$ is asymptotically given by:

$$P_{FA} \rightarrow_{N\rightarrow\infty} e^{-\lambda/2},$$

irrespective of the unknown disturbance pdf p_C. Moreover,

$$P_D(P_{FA}) \rightarrow_{N\rightarrow\infty} Q_1\left(\frac{\sqrt{2}|\bar{\alpha}|||\mathbf{v}||^2}{\sqrt{\mathbf{v}^H\boldsymbol{\Gamma}\mathbf{v}}}, \sqrt{-2\ln P_{FA}}\right),$$

where $Q_1(\cdot,\cdot)$ is the Marcum Q function of order 1

- The minimum number N of virtual spatial DoF needed to well-approximate the asymptotic performance defines the **massive MIMO regime**.

A comparison with the AMF $\Lambda_{\mathsf{AMF}}(\mathbf{x})$ [14]

$$\Lambda_{\mathsf{AMF}}(\mathbf{x}) = \frac{|\mathbf{v}^H \widehat{\mathbf{C}}^{-1}\mathbf{x}|^2}{\mathbf{v}^H \widehat{\mathbf{C}}^{-1}\mathbf{v}}, \quad \Lambda_{\mathsf{RW}}(\mathbf{x}) = \frac{2|\mathbf{v}^H \mathbf{x}|^2}{\mathbf{v}^H \widehat{\boldsymbol{\Gamma}}_l \mathbf{v}}.$$

- Multi-snapshots vs. Single-snapshot
  - $\Lambda_{\mathsf{AMF}}(\mathbf{x})$ requires a set of homogeneous secondary snapshots to get the full rank estimation $\widehat{\mathbf{C}}$ of $\boldsymbol{\Gamma}$,
  - $\Lambda_{\mathsf{RW}}(\mathbf{x})$ relies on a single spatial snapshot.
- Gaussian-based vs. Robust
  - $\Lambda_{\mathsf{AMF}}(\mathbf{x})$ is a CFAR detector only if $\mathbf{c}$ and the set of secondary data are Gaussian-distributed,
  - $\Lambda_{\mathsf{RW}}(\mathbf{x})$ is asymptotically CFAR for every disturbance vector $\mathbf{c}$ satisfying Assumption A1.

[14] F. C. Robey, D. R. Fuhrmann, E. J. Kelly, and R. Nitzberg, "A CFAR adaptive matched filter detector," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 28, no. 1, pp. 208–216, Jan 1992.

Numerical validation

We consider two different scenarios:

- Case 1: The disturbance is modelled as an AR(3) with

$$\bar{\boldsymbol{\rho}} = [0.5e^{j2\pi 0}, 0.3e^{-j2\pi 0.1}, 0.4e^{j2\pi 0.01}]^T,$$

- Case 2: The disturbance is modelled as an AR(6) with

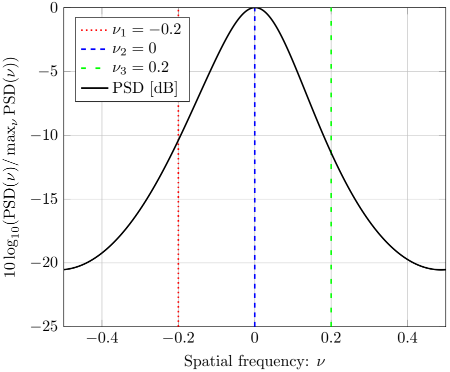
$$\bar{\boldsymbol{\rho}} = [0.5e^{-j2\pi 0.4}, 0.6e^{-j2\pi 0.2}, 0.7e^{j2\pi 0}, 0.4e^{j2\pi 0.1}, \\ 0.5e^{j2\pi 0.3}, 0.6e^{j2\pi 0.35}]^T,$$

- In both cases, the innovations $\{w_n, \forall n\}$ share a complex t-distribution:

$$p_w(w_n; \lambda, \eta) = (\sigma_w^2 \pi)^{-1} \lambda (\lambda/\eta)^{\lambda} (\lambda/\eta + |w_n|/\sigma_w^2)^{-(\lambda+1)}$$

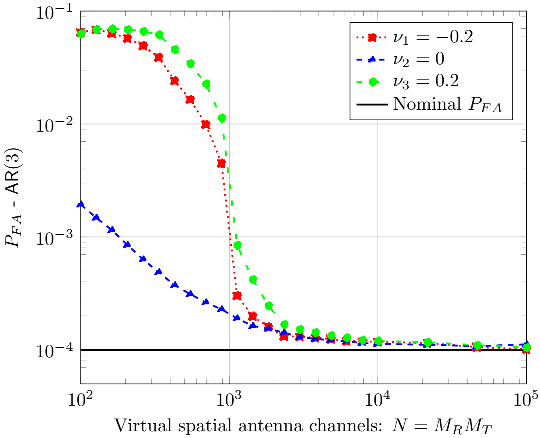
where $\lambda = 2$, $\sigma_w^2 = 1$ and $\eta = \lambda/\sigma^2(\lambda - 1)$.

Power Spectral Density (PSD) of the AR(3)



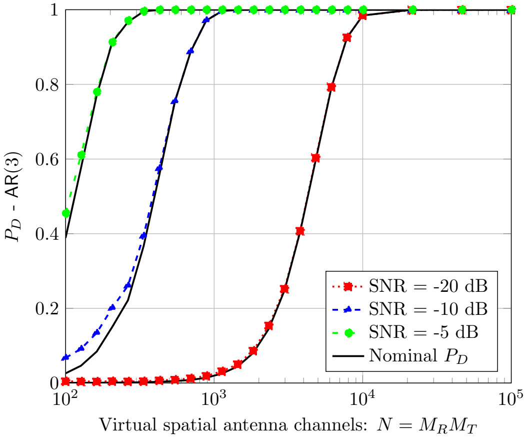
- Virtual steering vectors: $[\mathbf{v}_i]_n = e^{j\pi(n-1)\sin(\phi_i)}$, $n = 1, \ldots, N$ and $\phi_i = \arcsin(\nu_i/2)$ where $\nu_1 = -0.2$, $\nu_2 = 0$ and $\nu_3 = 0.2$.
- $\mathbf{S} = \mathbf{I}_{M_T}$ and $\mathbf{W} = \mathbf{I}_{M_T}$.

Estimated and theoretical P_{FA}: case 1



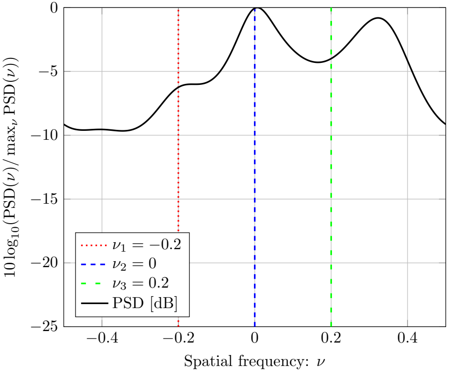
- The estimated P_{FA} tends to the nominal value $\overline{P_{FA}} = 10^{-4}$,
- The massive MIMO regime is achieved for $N = M_R M_T \geq 10^4$.

Estimated and theoretical P_D: case 1



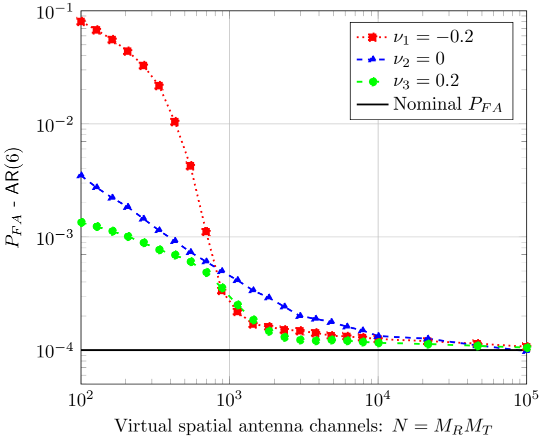
- The SNR is defined as $\mathrm{SNR} \triangleq 10 \log_{10}(|\bar{\alpha}|^2/\sigma^2)$.
- The estimated P_D is close to the asymptotic approximation.

Power Spectral Density (PSD) of the AR(6)



- Virtual steering vectors: $[\mathbf{v}_i]_n = e^{j\pi(n-1)\sin(\phi_i)}$, $n = 1, \dots, N$ and $\phi_i = \arcsin(\nu_i/2)$ where $\nu_1 = -0.2$, $\nu_2 = 0$ and $\nu_3 = 0.2$.
- $\mathbf{S} = \mathbf{I}_{M_T}$ and $\mathbf{W} = \mathbf{I}_{M_T}$.

Estimated and theoretical P_{FA}: case 2



- The estimated P_{FA} tends to the nominal value $\overline{P_{FA}} = 10^{-4}$,
- The massive MIMO regime is achieved for $N = M_R M_T \geq 10^4$.

On the P_D maximization

- The proposed robust Wald-type test has the CFAR property with respect to the unknown disturbance distribution.

- What about the Probability of Detection (P_D)? Can we maximize it somehow?

- From the previous results, we have that:

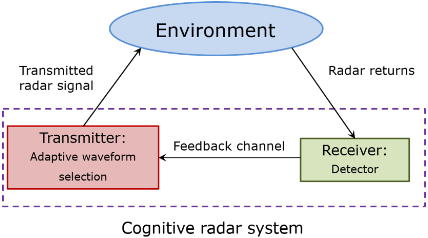
$$P_D(\lambda) \to_{N\to\infty} Q_1\left(\sqrt{\varsigma}, \sqrt{\lambda}\right),$$

$$\varsigma = \frac{2|\bar{\alpha}|^2 M_R^2 \|(\mathbf{WS})^T \mathbf{a}_T(\bar{\phi})\|^4}{\mathrm{tr}\left(\boldsymbol{\Gamma}\left[(\mathbf{WS})^T \mathbf{a}_T(\bar{\phi})\mathbf{a}_T^H(\bar{\phi})(\mathbf{WS})^* \otimes \mathbf{a}_R(\bar{\phi})\mathbf{a}_R^H(\bar{\phi})\right]\right)}.$$

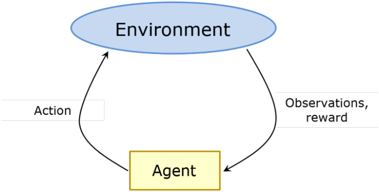
- We can maximize P_D by choosing a suitable waveform matrix **W** according to the observed scenario!

Cognitive Radar and Reinforcement Learning

Cognitive radar



Reinforcement Learning

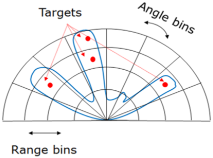


RL for Multi-target Detection in MIMO radar

- The radar acts as an agent continuously sensing the unknown environment (i.e., targets and disturbance).

- The agent evaluates its action using two types of information:
  - The state: the number of target),
  - The reward: the Probability of Detection (P_D).

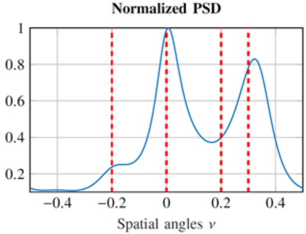
- The goal is to maximize the P_D (i.e. reward) by choosing the best action that is the optimal waveform matrix $\mathbf{W}$.[15]



[15] A. M. Ahmed, A. A. Ahmad, S. Fortunati, A. Sezgin, M. S. Greco and F. Gini, "A Reinforcement Learning Based Approach for Multitarget Detection in Massive MIMO Radar," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 57, no. 5, pp. 2622-2636, Oct. 2021.

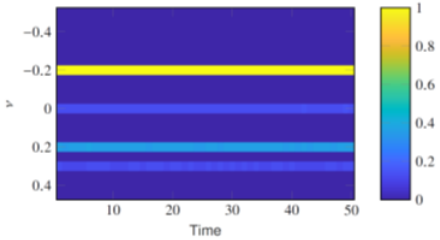
Some results: static targets (1/2)

- Four targets with different Signal to Noise ratio (SNR):
  - T1: $\nu = -0.2$, SNR = -5dB,
  - T2: $\nu = 0$, SNR = -8dB,
  - T3: $\nu = 0.2$, SNR = -10dB,
  - T4: $\nu = 0.3$, SNR = -9dB.
- The disturbance is modelled as an AR(6).
- The innovations w_n share a complex t-distribution:

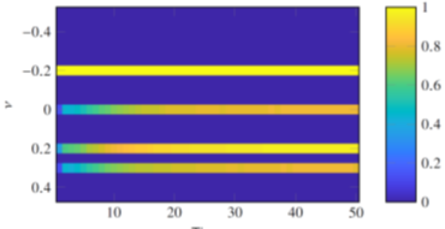


Some results: static targets (2/2)

- Omnidirectional beamforming ($\mathbf{W} = \mathbf{I}$):

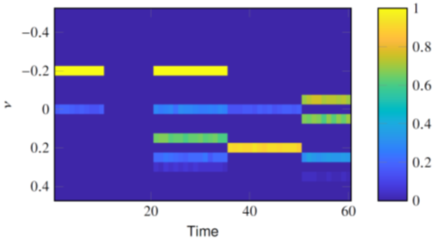


- RL-based beamforming ($\mathbf{W} = \mathbf{W_{RL}}$):

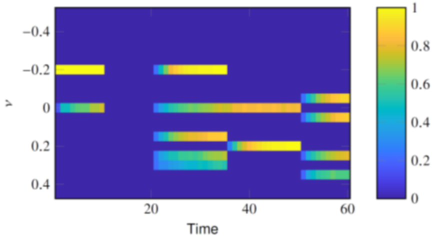


Some results: dynamic targets

- Omnidirectional beamforming ($\mathbf{W} = \mathbf{I}$):



- RL-based beamforming ($\mathbf{W} = \mathbf{W}_{\mathbf{RL}}$):



Concluding remarks

- By exploiting the increased number N of spatial DoF that a co-located MIMO radar can provide, a robust Wald-type detector Λ_{RW} is proposed.
- As $N = M_R M_T \to \infty$ and if the disturbance ACF decays at least polynomially fast, the asymptomatic distribution of Λ_{RW} does not depend on the *unknown* disturbance pdf.
- This represents a first attempt to apply the "massive" MIMO paradigm of communication systems to radar applications.
- Ongoing works:
  - Reinforcement Learning (RL) in dynamic environments,
  - Statistical optimality and robustness of RL procedures for target detection.