

Errata corrigé for the paper:

# Semiparametric CRB and Slepian-Bangs formulas for Complex Elliptically Symmetric Distributions

**Stefano Fortunati, Fulvio Gini, Maria S. Greco, Abdelhak M. Zoubir,  
Muralidhar Rangaswamy**

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List of typos in [1]:

1. In Eq. (17),  $\text{vecs}(\boldsymbol{\Sigma}_0)$  should be  $\text{vec}(\boldsymbol{\Sigma}_0)$ . Specifically, the correct version of Eq. (17) is:

$$\mathbf{s}_{\boldsymbol{\phi}_0} \triangleq \nabla_{\boldsymbol{\phi}} \ln p_Z(\mathbf{z}; \boldsymbol{\phi}_0, h_0) = [\mathbf{s}_{\boldsymbol{\mu}_0}^T, \mathbf{s}_{\boldsymbol{\mu}_0^*}^T, \mathbf{s}_{\text{vec}(\boldsymbol{\Sigma}_0)}^T]^T. \quad (17)$$

2. In the first line after Eq. (18),  $\mathbf{s}_{\text{vecs}(\boldsymbol{\Sigma}_0)}^T$  should be  $\mathbf{s}_{\text{vec}(\boldsymbol{\Sigma}_0)}^T$ .
3. A minus “-” is missing in front of the right-hand side of Eq. (25). The correct equation is:

$$\begin{aligned} \bar{\mathbf{s}}_{\text{vec}(\boldsymbol{\Sigma}_0)} &= d - \mathcal{Q}\psi_0(\mathcal{Q}) \times \\ &\times (\boldsymbol{\Sigma}_0^{-*/2} \otimes \boldsymbol{\Sigma}_0^{-1/2} \text{vec}(\mathbf{u}\mathbf{u}^H) - N^{-1} \text{vec}(\boldsymbol{\Sigma}_0^{-1})). \end{aligned} \quad (25)$$

4. A minus “-” is missing in front of  $\text{tr}(\mathbf{P}_i^0)$  in Eqs. (38), (40), (41), (42) . The correct equations are:

$$[\mathbf{s}_{\boldsymbol{\theta}_0}]_i \triangleq \frac{\partial \ln p_Z(\mathbf{z}; \boldsymbol{\theta}, h_0)}{\partial \theta_i} \Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0} = -\text{tr}(\mathbf{P}_i^0) + \psi_0(Q_0) \frac{\partial Q_0}{\partial \theta_i}, \quad (38)$$

$$\begin{aligned} [\mathbf{s}_{\boldsymbol{\theta}_0}]_i &= -\text{tr}(\mathbf{P}_i^0) - \psi_0(Q_0) (2\text{Re}[(\mathbf{z} - \boldsymbol{\mu}_0)^H \boldsymbol{\Sigma}_0^{-1} \boldsymbol{\mu}_i^0] + \\ &+ (\mathbf{z} - \boldsymbol{\mu}_0)^H \mathbf{S}_i^0 (\mathbf{z} - \boldsymbol{\mu}_0)), \quad i = 1, \dots, d. \end{aligned} \quad (40)$$

$$\begin{aligned} [\mathbf{s}_{\boldsymbol{\theta}_0}]_i &= d - \psi_0(\mathcal{Q}) \left( 2\sqrt{\mathcal{Q}} \text{Re} \left[ \mathbf{u}^H \boldsymbol{\Sigma}_0^{H/2} \boldsymbol{\Sigma}_0^{-1} \boldsymbol{\mu}_i^0 \right] + \right. \\ &\quad \left. + \mathcal{Q} \mathbf{u}^H \boldsymbol{\Sigma}_0^{H/2} \mathbf{S}_i^0 \boldsymbol{\Sigma}_0^{1/2} \mathbf{u} \right) - \text{tr}(\mathbf{P}_i^0) \\ &= -\psi_0(\mathcal{Q}) \left( 2\sqrt{\mathcal{Q}} \text{Re} \left[ \mathbf{u}^H \boldsymbol{\Sigma}_0^{-1/2} \boldsymbol{\mu}_i^0 \right] + \mathcal{Q} \mathbf{u}^H \mathbf{P}_i^0 \mathbf{u} \right) + \\ &\quad - \text{tr}(\mathbf{P}_i^0), \quad i = 1, \dots, d. \end{aligned} \quad (41)$$

$$\begin{aligned}
[\Pi(\mathbf{s}_{\theta_0} | \mathcal{T}_{h_0})]_i &= E_{0|\sqrt{\mathcal{Q}}} \{ [\mathbf{s}_{\theta_0}]_i | \sqrt{\mathcal{Q}} \} \\
&=_d -\text{tr}(\mathbf{P}_i^0) - 2\sqrt{\mathcal{Q}}\psi_0(\mathcal{Q})\text{Re} \left[ E\{\mathbf{u}\}^H \boldsymbol{\Sigma}_0^{-1/2} \boldsymbol{\mu}_i^0 \right] \\
&\quad - \mathcal{Q}\psi_0(\mathcal{Q})\text{tr}(\mathbf{P}_i^0 E\{\mathbf{u}\mathbf{u}^H\}) \\
&= -\text{tr}(\mathbf{P}_i^0) - N^{-1}\mathcal{Q}\psi_0(\mathcal{Q})\text{tr}(\mathbf{P}_i^0), \quad i = 1, \dots, d.
\end{aligned} \tag{42}$$

## References

- [1] S. Fortunati, F. Gini, M. S. Greco, A. M. Zoubir, and M. Rangaswamy, “Semiparametric crb and slepiian-bangs formulas for complex elliptically symmetric distributions,” *IEEE Transactions on Signal Processing*, vol. 67, no. 20, pp. 5352–5364, 2019.