Control of open quantum systems

Quantum control is a multidisciplinary field which has an immense ability to be applied in different fields such as chemistry, biology, mathematics, engineering, physics, etc. Different objectives can be considered such as state transfer, stabilization of quantum states, robust quantum control, etc. The ability to control quantum systems will be a critical step in realizing quantum technologies.

Two main approaches for quantum feedback are considered *measurement-based feedback* and *coherent feedback (autonomous feedback)*. The idea of *quantum measurement-based feedback* is in principle similar to that of classical feedback control, as the control input is a classical signal. However, being the system to be controlled a quantum system, the state is subject to quantum measurement back-action; see Figure 1. This means that by measuring a quantum system, we change the actual state of the quantum system - a feature that has no classical counterpart. A major difficulty with implementing measurement-based feedback is that the slowest time scale is set by the classical processing of information (typically digitally) by the quantum filter meaning that the feedback law is too slow for the system time scale. Historically, such kinds of controllers (involving filtering and feedback) are implemented in an analogue way via classical electronic circuits [10].

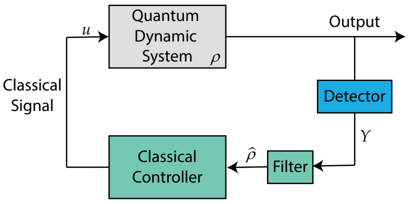

Figure 1: Measurement-based feedback.

Quantum measurements have probabilistic nature. Hence, when dealing with quantum feedback control, the evolution of the system is described by nonlinear stochastic equations. Here we will study control of open quantum systems by applying quantum feedback. This can be formulated as a stochastic non-linear control problem: a quantum filtering problem and a state feedback control problem for the filter. Open quantum systems are quantum systems in interaction with an environment. This interaction perturbs the system states and causes loss of information from the system to the environment. However by applying quantum feedback control, the system can ”fight" against this loss of information.

Typical applications of quantum feedback control appear in stabilization of pure states and control of non-classical states such as squeezed or entangled states. The experiment realized in 2011 by Serge Haroche (Nobel prize of Physics, 2012) group [14] is a relevant example where a real-time feedback control is applied to stabilize an arbitrary photon number state in a microwave cavity. This was the first realization of a real-time feedback which stabilizes particular photon number states.

This PhD thesis is multidisciplinary between control theory, mathematical physics, probability and quantum physics. In this thesis, we consider Markov models to express the quantum system and we will study quantum measurement-based feedback to stabilize open quantum systems. Feedback stabilization of pure states has received particular interest [14, 11, 17] in developing quantum technologies. The evolution of an open quantum system is described by Stochastic Master Equations (SMEs). We will study feedback strategies which stabilize the quantum system towards a predetermined pure state or pure subspace by using especially stochastic Lyapunov functions. For this, we will apply stochastic control theory [3], stochastic stability [3], stabilization of quantum systems using non-linear control tools (see e.g., [16, 2, 3, 11, 18, 17, 5]).

Our objective here is to develop systematic and robust strategies for quantum feedback design applied to any open quantum systems. In real experiments different sorts of imperfections may be present, as for instance inefficient detectors, unknown initial states, imprecise knowledge of the detector efficiency and other physical parameters, delays, etc. Hence, from a practical point of view, choosing feedback controls which are robust to such imperfections is an important, and challenging, problem. Our preliminary results on feedback stabilization of some particular open quantum systems can be found in [4, 5, 6, 7, 8]. In this thesis, we will prove robustness with such imperfections. Also, the filter equations could be high dimensional. Our aim is to provide new approaches to design reduced quantum filters (of low dimensions) and apply them in feedback control of open quantum systems. This has a significant impact in engineering of quantum devices because one reduces the complexity of the quantum dynamical systems.

Quantum feedback plays a major role in quantum information processing [13, 15, 1, 9]. Feedback can help to fight against the decoherence induced by the environment and against the error in the hardware system. In the second part of the thesis, we will study applications of stabilizing feedback control strategies in different contexts [1, 12].

During this thesis, we will collaborate with the leading experimentalists and theoreticians who are members of this project (https://anr.fr/Projet-ANR-19-CE48-0003). Different missions are envisaged for the hired PhD student.

Practical informations on the application:

- This PhD thesis will be financed by Agence Nationale de la Recherche (ANR).
- Laboratory: L2S (Laboratoire des Signaux et Systèmes), UMR 8506 Université Paris-Saclay, CNRS, CentraleSupélec, 91190 Gif-sur-Yvette, France.
- We will receive the applications until the position is filled (preferably before the end of August) and will review the application from the beginning of July.
- The start date is in fall 2021/winter 2022.
- We are looking for a talented candidate who is passionate about research on such an interdisciplinary field, with a very good english skill and a master (or equivalent) degree in applied mathematics, mathematics, control theory/engineering, mathematical physics.
- Please send a detailed CV including the list of master courses and projects you have worked on with brief descriptions of your contributions, academic record, a motivation letter and contact details of two or three references, to Nina H. Amini (nina.amini@centralesupelec.fr).
- The academic excellence is the only criterion for selection.

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