

A control theory view on self-organized criticality

Contact: luca.greco@centralesupelec.fr, antoine.chaillet@centralesupelec.fr

Abstract

Self-organized criticality is a widespread phenomenon in which a dynamical system adjusts certain of its parameters in order to operate at the boundary between two distinct behaviors. From a control theory perspective, this phenomenon can be interpreted as the autonomous regulation of a parameter in the vicinity of a bifurcation point. Whether certain brain structures self-organize around criticality remains a subject of ongoing debate in neuroscience. The objective of this PostDoc proposal is to understand which control strategies may give rise to self-organized criticality, both within a general framework and in models of neuronal populations.

Context

Self-organized criticality (SOC) is a phenomenon observed in many physical and biological systems. It characterizes dynamical systems that autonomously evolve toward a critical equilibrium, that is, a state located at the boundary between two qualitatively distinct behaviors. In other words, SOC corresponds to a situation in which one or more parameters self-regulate toward a bifurcation point. SOC plays a crucial role in the emergence of complexity, since in such systems, small perturbations can generate radically different responses. A classic example of SOC is a sandpile to which grains are added one by one: the pile eventually reaches a critical state, and the addition of a single grain may (or may not) trigger an avalanche of varying magnitude.

Theoretical studies [Beggs 2007; Hesse and Gross 2014] and experimental work [Beggs and Plenz 2003] suggest that the brain may operate near criticality, which could contribute to its optimal computational capabilities relative to its metabolic constraints. Indicators of SOC have been identified in in vitro neuronal cultures as well as in the brains of curarized animals. Despite these findings, the hypothesis of a brain operating near criticality remains highly debated [Touboul and Destexhe 2017; Destexhe and Touboul 2021]. The main reason for this debate is that markers traditionally associated with critical systems (such as the presence of avalanches or power-law distributions of event occurrences) can also emerge in systems operating far from criticality.

Objectives

One of the main challenges in the current study of self-organized criticality lies in the reliance on high-dimensional models, whose dynamics are simulated and subsequently analyzed using statistical tools. Due to their dimensionality, these models are poorly suited to a formal analysis of their dynamical properties, such as structural stability and criticality in the sense of bifurcation theory.

A first step toward addressing the question on presence or absence of self-organized criticality in the brain, is to employ low-dimensional (possibly deterministic) neuronal population models and to investigate the feedback mechanisms that may lead the system to self-regulate around a bifurcation point. Our objective here is to treat the bifurcation parameter as a dynamical variable and to determine (biologically plausible) control strategies that enable it to converge toward its (a priori unknown) critical value. This control-theoretic perspective on self-organized criticality has received limited attention so far, with the notable exception of [Moreau and Sontag 2003], which, however, is restricted to very specific classes of systems. The difficulty stems from the intrinsically nonlinear nature of the dynamics involved, as well as from the non-standard nature of the control objective, which cannot be framed as either stabilization or trajectory tracking.

Beyond the design of such control strategies, this PostDoc aims to address fundamental questions of interest to other disciplines, in particular the minimal information required for self-organized criticality. In control-theoretic terms, this question is closely related to observability issues.

One of the most ambitious directions we want to pursue in this PostDoc is to reconcile the two points of view on criticality. From one side, the empirical definition based on the statistical signatures associated to avalanches coming from statistical physics and neuroscience, on the other side, the definition based on bifurcation and structural stability coming from systems theory.

Real cortical neural networks are large, extended systems, while the mean-field approximation is a macroscopic description at the level of neuronal population. The order and the control parameters are global (emergent) quantities. If the brain is critical and a mechanism is responsible for its self-regulation, then such mechanism must act on a local scale. In other words, a neuron has no access to the global order parameter but only to the state of a limited number of neurons connected to it. The question is: how can we adapt a mechanism developed for a mean-field model to work on a microscopic scale on the basis of only local measurements?

Moreover, if such adaptation succeeds, the large system should generate avalanches. How many signatures of criticality can in this case be measured in simulation? Are they consistent with those experimentally measured in real networks?

Methods

The proposed approach is grounded in control theory and dynamical systems theory. Core concepts from control theory, such as feedback, observers, and adaptive control, will be considered. Bifurcation analysis, along with concepts related to structural stability from dynamical systems theory, will also play a fundamental role.

Profile and skill required

Strong background on automatic control or in mathematics. Basic knowledge on stability analysis for nonlinear systems. Interest for dynamical systems and neurosciences.

References

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