Title: Practical control of networks of hyperbolic systems

Laboratory: Laboratoire des signaux et systèmes, CNRS, CentraleSupélec, Université Paris Saclay, 91190, Gif-sur-Yvette, France

PhD Advisors: Jean AURIOL, Lucas BRIVADIS[1].

Description of the Thesis

I. Context and scientific objectives

Distributed parameter systems provide a natural representation of industrial processes involving the evolution of quantities in time and space. In particular, hyperbolic partial differential equations play a crucial role in the mathematical description of transport phenomena with finite propagation speeds, e.g., transport of matter, sound waves, and information. **Networks of hyperbolic Partial Differential Equations (PDEs) systems**, possibly coupled with Ordinary Differential Equations (ODEs), constitute an essential paradigm to describe a wide variety of large complex systems, including wave propagation, traffic network systems, electric transmission lines, hydraulic channels, drilling devices, communication networks, smart structures, multiscale and multiphysics systems [1, 2, 3]. Controlling and monitoring networks of hyperbolic systems are **difficult control engineering problems** due to the **distributed** nature of the different subsystems composing the network (time and space dependency), the possibly involved **graph structure** of the network, and the physical/economic **infeasibility of placing sensors and actuators everywhere** along the spatial domain. The stringent operating, environmental, and economical requirements and the high mathematical complexity of these systems explain why traditional control methods exhibit a limited range of applicability and have not been successful at high technology readiness levels (TRLs) [4, 5]. Thus, the theory of control of distributed parameter systems needs substantial advancements to achieve control and estimation objectives for such network structures.
A relevant example of a network of hyperbolic systems is provided by traffic networks. Controlling traffic networks is essential in the near future for reducing contamination and fluidifying the density of cars on the roads. The traffic on a single freeway segment can be modeled by a set of hyperbolic equations (known as the ARZ model [3]), and stabilizing controllers that suppress stop-and-go oscillations have been designed in [3, 6]. However, when considering a general freeway network configuration, there is an effect on traffic flow stability from freeway branches merging or diverging, from branches looping back or forming a "beltway." Therefore, the **controllability of general networks** of ARZ models, with inputs at various locations along the interconnected freeway branches, is a complex question.

In this thesis, we consider networks composed of interconnected *elementary* hyperbolic subsystems. These different hyperbolic subsystems correspond to one-dimensional linear balance law systems [1, Chap. 5]. They are called *elementary* in the sense that when taken alone, we know how to design stabilizing control laws. The different subsystems are connected through their boundaries. Examples of possible network configurations are given in Figure 1. Thus, a network of hyperbolic systems can be described as a **graph**. For instance, each elementary hyperbolic subsystem can be identified with an edge of a given graph. At the same time, interactions between the PDEs occur at the graph's vertices. Such a **graph representation** has been used in [7] to describe networks of wave equations.

Thesis goal: **The general objective** of this thesis is to develop a **systematic framework** for the **practical control** of **networks of linear hyperbolic systems**. The proposed control strategies will have to be constructive and easily implementable. In particular, we want to answer the following two problems.

1. Given a configuration of actuator/sensor, we want to verify that this configuration makes the system controllable/observable before designing an appropriate control law.

2. Considering a given number of actuators, we aim to find admissible locations to guarantee controllability/observability.

[1] The advisors are with Université Paris-Saclay, CNRS, CentraleSupélec, Laboratoire des signaux et systèmes, 91190, Gif-sur-Yvette, France. Emails: {jean.auriol,lucas.brivadis}@centralesupelec.fr.

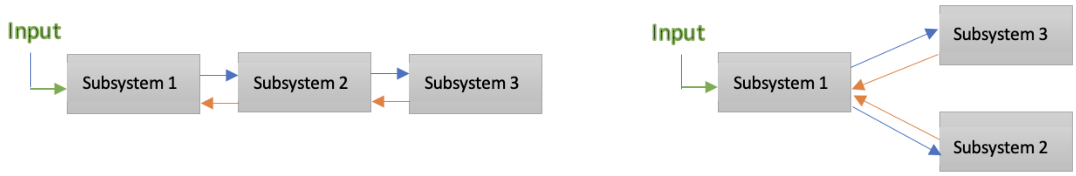

Figure 1: Examples of possible network configurations: chain structure (left), junction/tree (right). The arrows picture possible boundary couplings between the subsystems.

II. Scientific approach

To solve these challenging open questions, we aim to use an innovative methodology that combines promising and complementary techniques, which have been developed for simple network structures [8]. More precisely, our methodology relies on the **theory of integral equations**. Using an appropriate **integral backstepping transformation** [9], it is possible to rewrite the original hyperbolic network as a set of simpler **Integral Delay Equations** (IDEs) with pointwise and distributed control terms [8, 10]. For such systems, spectral controllability conditions can easily be obtained. These conditions have been shown to be sufficient to design stabilizing controllers/observers for systems in abstract form [11, 12] and for test cases [8]. More precisely, stabilizing controllers are obtained by solving a set of **Fredholm integral equation**. The existence of a solution is implied by the controllability of the system.

To extend this approach to *more involved network structures*, we will use **graph theory** to simplify the network's description and subdivide it into simpler sub-networks. Indeed, the structure of the IDE system is related to the properties of the graph describing the networks (e.g., number of cycles, incidence matrix). Using the concept of structural controllability [13], we will identify reflections of graph-theoretic notions on the system properties and relate the graph structure of the network with the proposed IDE representation. We can use these properties to design appropriate stabilizing control laws. In this context, it appears essential to simplify the design by subdividing the graph into sub-graphs. Note that this approach can be leveraged to obtain numerical solutions to the problem. State observers will be designed following a similar path.

The scientific objective of the thesis is to first extend these previous approaches, developed for simple network configurations, to a more general framework. The principal steps of the proposed work are listed as follows:

- For an arbitrary network of hyperbolic systems with a **given configuration** of sensors and actuators, we want to obtain conditions that characterize the controllability/observability of the system. In particular, we aim to identify reflections of graph-theoretic notions on the system properties to simplify the controllability conditions and, therefore, the design of the corresponding controllers.
- Once we have characterized the controllability/observability of the network, we aim to design stabilizing output-feedback control laws using our methodology based on IDEs. As this objective is highly ambitious, we will start by considering a gradation in the complexity of the network by focusing on **specific network configurations**: chain, divergence, star, simple trees, one cycle. We plan to tackle the general problem very gradually, thus obtaining many intermediate results on specific cases that would prove deeply interesting due to the scarcity of literature on this topic.
- For a given network of hyperbolic systems, find the minimum number of actuators/sensors (and their respective position in the network) to guarantee the possibility of controlling/observing. Then, assuming we now have a fixed number of actuators/sensors greater than this value, we want to know all the **admissible configurations** under which it is possible to design output-feedback controllers. We expect to connect the minimal number of actuators/sensors with some network graph properties (e.g., number of cycles, branches). Again, we will simplify the analysis by considering first specific network configurations with scalar subsystems.
- Finally, we plan to showcase the efficiency of the proposed approaches through numerical simulations on specific academic examples (for instance, on (traffic regulation). We also aim to o **deploy, demonstrate, and validate** the techniques on an experimental setup already existing in L2S, for the **active control of vibrations in mechanical structures**. This test case corresponds to a thin mechanical beam with one clamped edge and is equipped with piezoelectric actuators and sensors. The application purpose is similar to industrial issues, such as active damping of onboard optical and/or electronic equipment or the control of micro-endoscopes actuated with electro-active

polymers.

III. Potential international collaborations

The proposed project will be linked to the International Research Network (CNRS) "PHESTINS" (in collaboration with the United States and Canada), a consortium starting in the following months. Collaborations with the University of California San Diego or the Hong Kong University of Science and Technology could also be considered to benchmark the proposed strategies on the problem of traffic networks. Visiting periods in these universities may therefore be considered.

IV. Required skills

This thesis topic mainly requires good skills in control systems and mathematics (Grandes Ecoles or Master in mathematics/control). Very good results in the engineering curriculum as well as expertise in the topics related to automatic and partial differential equations, will constitute strengths to the proposed subject. The proposed subject shall lead to the acquisition of strong theoretical skills in the field of control of systems described by partial differential equations. In particular, the candidate shall become familiar with the modeling of dynamical systems, with control design, hyperbolic equations, graph theory, and numerical or experimental implementation. The candidate shall also become familiar with Julia, Matlab, or Python (numerical methods, simulations).

V. Application

To apply, write an email with your CV and a transcript to J. Auriol and L. Brivadis. The thesis should start in October 2024.

VI. References

[1] G. Bastin and J.-M. Coron. *Stability and boundary stabilization of 1-d hyperbolic systems*. Springer, 2016.

[2] U. J. F. Aarsnes and R. Shor. Torsional vibrations with bit off bottom: Modeling, characterization and field data validation. *Journal of Petroleum Science and Engineering*, 163:712–721, 2018.

[3] H. Yu and M. Krstic. *Traffic Congestion Control by PDE Backstepping*. Springer, 2023.

[4] J. Auriol and F. Di Meglio. Robust output feedback stabilization for two heterodirectional linear coupled hyperbolic PDEs. *Automatica*, 115:108896, 2020.

[5] R. Curtain and H. Zwart. *An introduction to infinite-dimensional linear systems theory*, volume 21. Springer, 2012.

[6] L. Zhang, C. Prieur, and J. Qiao. PI boundary control of linear hyperbolic balance laws with stabilization of arz traffic flow models. *Systems & Control Letters*, 123:85–91, 2019.

[7] R. Dager and E. Zuazua. *Wave propagation, observation and control in 1-d flexible multi-structures*, volume 50. Springer Science & Business Media, 2006.

[8] J. Redaud, J. Auriol, and S.-I. Niculescu. Stabilizing output-feedback control law for hyperbolic systems using a fredholm transformation. *IEEE Transactions on Automatic Control*, 2022.

[9] M. Krstic and A. Smyshlyaev. *Boundary control of PDEs: A course on backstepping designs*. SIAM, 2008.

[10] J. Auriol and F. Di Meglio. An explicit mapping from linear first order hyperbolic PDEs to difference systems. *Systems & Control Letters*, 123:144–150, 2019.

[11] L. Brivadis, V. Andrieu, U. Serres, and J.-P. Gauthier. Luenberger observers for infinite-dimensional systems, back and forth nudging, and application to a crystallization process. *SIAM Journal on Control and Optimization*, 59(2):857–886, 2021.

[12] N. Vanspranghe and L. Brivadis. Output regulation of infinite-dimensional nonlinear systems: A forwarding approach for contraction semigroups. *SIAM Journal on Control and Optimization*, 61(4):2571–2594, 2023.

[13] C.-T. Lin. Structural controllability. *IEEE Transactions on Automatic Control*, 19(3):201–208, 1974.

Advisors' Biographies

Jean AURIOL (thesis supervision load: 50%). He received his Master's degree in civil engineering in 2015 (major: applied maths) from MINES ParisTech, part of PSL Research University, and in 2018 his PhD degree in control theory and applied mathematics from the same university (Centre Automatique et Systèmes). His Ph.D. thesis, titled *Robust design of backstepping controllers* *for systems of linear hyperbolic PDEs*, has been nominated for the best thesis award given by the GDR MACS and the Section Automatique du Club EEA in France. From 2018 to 2019, he was postdoctoral researcher at the Department of Petroleum Engineering, University of Calgary, Canada, where he was working on implementing backstepping control laws to attenuate mechanical vibrations in drilling systems. Since December 2019, he has been an associate researcher (Chargé de Recherche) at CNRS, Université Paris-Saclay, CentraleSupélec, L2S. His research interests include robust control of hyperbolic systems, neutral systems, networks, and interconnected systems. In particular, during the last years, he has worked on the design of stabilizing control laws for interconnected PDE-ODE systems, derived the *interconnected recursive framework methodology* and introduced a new approach based on Fredholm transforms. These new tools will be key to dealing with networks of PDE systems.

Lucas BRIVADIS (thesis supervision load: 50%). He received in 2018 his engineering degree from École Centrale de Lyon and his master's degree in applied mathematics from Université Lyon 1. He defended his PhD in 2021 at LAGEPP, Université Lyon 1. From 2021 to 2022, he was a postodcotral researcher at L2S, (CNRS, CentraleSupélec, Université Paris-Saclay). Since 2022, he is a CNRS researcher (chargé de recherche) at L2S. His research interests focus on infinite-dimensional observer design for hyperbolic PDEs under weak observability assumptions (with applications to crystallization processes) and output feedback stabilization of nonlinear systems with observability singularities.